

**FOTOFISSION AV ^{236}U OCH ^{238}U
VID LÅGA ENERGIER**

Anders Alm



Lund 1975

FOTOFISSION AV ^{236}U OCH ^{238}U VID LÅGA ENERGIER

Av

Anders Alm

Fil. kand., Sm

Akademisk avhandling som för avläggande av filosofie
doktorsexamen vid matematisk-naturvetenskapliga
fakulteten vid universitetet i Lund kommer att offent-
ligen försvaras i fysiska institutionens föreläsnings-
sal B fredagen den 7 november 1975 kl 10.15.

"That was the phoniest
story I ever heard."

G. Marx

I. An experimental study of mass asymmetry in subbarrier
photofission of ^{238}U (Liljenström and T. Kivikangas).
Nuclear physics A 215 (1973) 461

II. Subbarrier photofission of ^{238}U (Liljenström and
T. Kivikangas och L. J. Lindgren).
Physics and chemistry of Fission 1973 vol. 1
Proceedings of a symposium, Rochester, New York
13-17 Aug 1973 (IAEA, Vienna 1974)

III. Fission fragment angular distributions and yields
of ^{236}U in low energy photofission
FOTOFISSION AV ^{236}U OCH ^{238}U
(Liljenström och L. J. Lindgren)

Nuclear physics Report LUNP 7585
VID LÅGA ENERGIER

Preliminary Report vid:

1) The International Conference on Photonnuclear
Reactions and Applications, 28-30 March 1972,
Anilone Conference Ground/Pacific Grove/California

2) The third symposium on the Physics and Chemistry
of Fission, 13-17 Aug 1973, Rochester, New York

Anders Alm

Fil.kand., Sm

"That was the phoniest
story I ever heard."
G. Marx

FOTOFISSION AV 236 U OCH 238 U

VID LÅGA ENERGIER



58

Anders Alm
Fil.Kand., Sm

Föreliggande avhandling omfattar följande arbeten:

- I. An experimental study of mass asymmetry in subbarrier photofission of ^{238}U (tillsammans med T Kivikas).

Nuclear physics A 215 (1973) 461

- II. Subbarrier photofission of ^{238}U (tillsammans med T Kivikas och L J Lindgren).

Physics and chemistry of Fission 1973 vol. 1

Proceedings of a symposium, Rochester, New York

13-17 Aug 1973 (IAEA, Vienna 1974)

- III. Fission fragment angular distributions and yields of ^{236}U in low energy photofission

(tillsammans med L J Lindgren)

Nuclear physics Report LUNP 7506

Preliminära rapporter gavs vid:

- 1) The international Conference on Photonuclear Reactions and Applications, 26-30 March 1973, Asilomar Conference Ground/Pacific Grove/California
- 2) The third symposium on the Physics and Chemistry of Fission, 13-17 Aug 1973, Rochester, New York

I. Inledning.

De arbeten, som beskrivs i ovanstående uppsatser har alla utförts inom ramen för en forskningsverksamhet knuten till institutionen för kärnfysik i Lund. Denna forskningsverksamhet bedrivs vid den elektronaccelerator av mikrotrontyp, som byggts på institutionen för elektronfysik vid Tekniska Högskolan i Stockholm. Arbetena skildrar några aspekter på fission av urankärnor, som exciterats genom växelverkan med högenergetisk elektromagnetisk strålning, gammastrålning. Den använda acceleratoren ger gammastrålning med kontinuerlig energifördelning, så kallad bromsstrålning, genom att strålen av accelererade elektroner träffar ett strålmål av platina. Bromsstrålningens energi är ~ 6 MeV. Eftersom bromsstrålning allmänt kan produceras med energi upp till 1000 MeV, karakteriseras den i arbetena använda som lågenergetisk.

II. Kärnfysikalisk bakgrund.

Fissionsprocessen upptäcktes av Hahn och Strassmann år 1939¹⁾. Samma år publicerades den första teoretiska tolkningen av Bohr och Wheeler²⁾. Denna tolkning utgår från föreställningen om atomkärnan som en laddad vätskedroppe. I denna droppe dominerar två, varandra motverkande krafter: en ytspänning, som strävar att ge kärnan en sfärisk form, och en elektrostatisk repulsion, som strävar att deformera kärnan. Dessa krafters samspel ger i kärnan upphov till en deformationspotentiell energi, som har sitt maximum vid den deformation där den elektrostatiske repulsionen tar överhanden över ytspänningen (se figur).

Kvantmekaniskt kan visas att det finns en från noll skild sannolikhet för att en atomkärna skall klyvas, även om dess energi

är lägre än fissionsbarriärens höjd. Denna effekt kallas barriärpenetration, och är orsaken till spontan fission av tunga kärnor. Fissionssannolikhetens, "fissionsträffytans", energiberoende är vid excitationsenergier motsvarande höjden på fissionsbarriären avhängigt av denna höjd, och även av barriärens bredd. Dessa barriärparametrar kan bestämmas, om man ställer upp en modell för fissionsbarriären, i vilken man approximerar barriärens form på lämpligt sätt. Ofta används en parabelformad barriär, för vilken ett exakt uttryck för barriärpenetrationen som funktion av excitationsenergien existerar. I detta uttryck ingår de ovan nämnda parametrarna, och jämförelse mellan modell och experiment ger empiriska värden på dem.

Ehuru vätskedroppsmodellen beskriver kärnfysikaliska medelegenskaper över ett stort masstalsområde för kärnan, så ger den felaktiga förutsägelser inom vissa områden. Sålunda ger vätskedroppsmodellen alltid en symmetrisk massfördelning av fissionsfragment vid fission av kärnor, medan massfördelningen vid fission av de tunga aktiniderna experimentellt är asymmetrisk. Ej heller ger vätskedroppsmodellen någon antydning om en förklaring till förekomsten av speciellt stabila nukleonkonfigurationer för de s.k. "magiska talen" (2, 8, 20, 28, 50, 82 och 126). Om proton-talet Z eller neutron-talet N är magiskt, så avviker kärnans egenskaper från de medelegenskaper som förutsägs av vätskedroppsmodellen.

Det framstod efterhand allt tydligare att de avvikelser från vätskedroppsmodellen som är förknippade med magiska tal kunde förklaras som skaleffekter, analoga med dem som uppträder i atomens elektronhölje. Skalmodellen formulerades av Haxel-Jensen-Suess³⁾ och Göppert-Mayer⁴⁾. Med denna modell kunde de magiska talen reproduceras. En utvidgning av skalmodellen, gjord av S.G. Nilsson⁵⁾, kunde förklara förekomsten av kärnor med

stabil grundtillstånd deformation, därmed antydande att skal-effekter är av betydelse inte endast vid sfäriska former på kärnan. En bekräftelse av detta, av speciell relevans för fissionsprocessen gjordes av Swiatecki⁶⁾ och Johansson⁷⁾, vilka visade att en förbättrad korrelation av systematiken hos livstider för spontan fission av tunga kärnor kunde uppnås, om den ur vätskedroppsmodellen erhållna fissionsbarriären korrigerades för skal-effekter i grundtillståndet.

Strutinski⁸⁾ angav år 1967 en modell, där skaleffekterna adderades som en korrektion till vätskedroppsmodellen. Med denna beräkningsmetod erhåller man de tidigare nämnda stabila grundtillstånd deformationerna för vissa kärnor. Vidare visade sig denna metod ge en skalkorrektion till fissionsbarriären. För kärnor i aktinidområdet erhöles i fissionsbarriären ett sekundärt minimum i närheten av sadelpunkten (se figur). Denna struktur i barriären gav en förklaring till tidigare kända, men hittills ej förklarade fenomen såsom resonanser i fissionsträffytan, vilka tolkades som vibrationstillstånd i den andra gropen, och fördröjd fission, som tolkades som fissionssönderfall från grundtillståndet i andra gropen.

Om man inkluderar strutinskikorrektionen vid en flerdimensionell undersökning av kärnans potentialenergiyta, där man tar hänsyn till flera deformationskoordinater, så visar det sig att den yttre sadelpunkten kan ha en stabil asymmetrisk deformation. Räkningar gjorda av flera olika grupper⁹⁻¹²⁾ ger entydigt att så är fallet för de tyngre aktiniderna. Trots svårigheten att följa systemet ända till den punkt där fragmenten separerar ger detta en antydning om orsaken till den asymmetriska fördelningen av fissionsfragmentens massor vid fission av aktiniderna.

År 1956 framkastade A. Bohr¹³⁾ hypotesen att lågenergifission kunde förklaras med hjälp av ett fåtal nivåer i den till sadelpunkten deformerade kärnan. Trots att nivåavståndet vid excitationens energin 6 MeV, i grundtillståndsdeformationen ("compound-kärnan") är ~ 1 eV, så är vid sadelpunkten större delen av denna energi bunden vid deformationen. Därför är kärnan termodynamiskt sval och kan förväntas ha ett spektrum av exciterade nivåer liknande dem en lågt exciterad kärna har vid grundtillståndsdeformationen. Denna hypotes bekräftas av en stor mängd experimentella data.

III. Fotofission av $^{236,238}\text{U}$.

I föreliggande avhandling studeras fotoinducerad fission av isotoperna $^{236,238}\text{U}$ i energiområdet 5-6.5 MeV. Eftersom fissionsbarriärens höjd ligger i detta energiområde förväntar man sig att Bohrs kanalthypotes skall vara giltig. Vidare bör, som tidigare antytts, träffytans energiberoende återspegla barriärens form. Träffytan beror ungefär exponentiellt på excitationens energin, och det fordras starka fotonkällor för att göra mätningar av de mycket låga fissionsträffytorna vid energier långt under barriärhöjden. Den starkaste källan för fotoner i detta energiområde är bromsstrålning. Man använder ett kontinuerligt spektrum av fotoner, $N(E, E_{\text{max}})$ upp till elektronens kinetiska energi E_{max} . Med denna metod erhålles ett integrerat fissionsutbyte

$$Y_f(E_{\text{max}}) = c \int_0^{E_{\text{max}}} \sigma_f(E) N(E, E_{\text{max}}) dE$$

Denna integralekvation måste lösas för att träffytan $\sigma_f(E)$ skall erhållas. Det visar sig uppträda numeriska svårigheter vid lösningen av ekvationen, vilket avsevärt försvårar jämförelsen med teori. En faktor som i vårt fall emellertid i viss mån under-

lättar tolkningen av utbyteskurvan är det starka energiberoendet i träffytan, vilket medför att endast ett begränsat energiområde strax under energin $E_{\max}$ bidrar i integraluttrycket ovan.

Fissionsprocessen uppdelas i två steg: excitation av kärnan-i vårt fall genom absorption av en foton- och det påföljande fissionssönderfallet av den bildade compoundkärnan.

Fotoabsorptionsträffytan i det studerade området är av elektrisk dipolkaraktär, med ett litet inslag av elektrisk kvadrupolabsorption. Förhållandet mellan dipol- och kvadrupolträffytorna är ca 0.02. Frånvaron av tillförlitliga experimentella data för energiberoendet av absorptionsträffytorna utgör en försvårande faktor vid analysen av fotoinducerade reaktioner i detta energiområde.

Vid fissionssönderfallet måste hänsyn tas till konkurrerande sönderfallsprocesser. De processer som kommer i fråga i det studerade energiområdet är gammadeexcitation och neutronemission. Bidraget från dessa processer kan i analysen beräknas ur välkända statistiska modeller.

I arbete I studeras utbytet av fissionsprodukter vid fotofission av ^{238}U . Energiberoendet för fissionsutbytet av ^{111}Ag och $^{115,117}\text{Cd}$, karakteristiska för symmetrisk fission, och ^{139}Ba , karakteristisk för asymmetrisk fission mättes. Isotoperna separerades med radiokemiska metoder och deras β -aktiviteter mättes. Förhållandet mellan de symmetriska och asymmetriska fissionsutbytena som funktion av excitationsenergin återspeglar variationen i dal-till-topp-förhållandet hos fissionsfragmentens massfördelning. Detta förhållande visar sig variera oerhört kraftigt med energin i det studerade massområdet. Från att vara $\sim 3 \cdot 10^{-4}$ mellan 5 MeV och 5.3 MeV stiger kvoten brant till $\sim 10^{-2}$ vid 5.9 MeV. Efter 6 MeV finns en tendens till ett avtagande.

Detta förlopp speglar en kraftig variation i det relativa bidrag till massfördelningen som den symmetriska fissionen ger.

Arbete II utgör ett försök att i en enkel modell tolka de experimentella resultaten från arbete I. Som ovan nämnts, ger beräkningar av kärnans energipotentialyta att den sadelpunkt, som svarar mot den yttre barriären, för ^{238}U har asymmetrisk form, dvs fissionsbarriären är lägst för en stabil asymmetrisk deformation, och har ett maximum i symmetririktningen. Dessa barriärformer användes av Tsang och Wilhelmy¹⁴⁾ för att beräkna massasymmetrin i fission, karakteriserad av topp-till-dal-förhållandet i massfördelningen. Sannolikheten för fission till asymmetriska och symmetriska produkter antogs vara proportionell mot nivåtätheten vid respektive barriär. Vid energier under fissionsbarriären ersattes nivåtätheten med penetrationssannolikheten genom respektive barriär. Det visade sig att korrelationen var påtaglig.

I arbete II beräknas dal-till-topp förhållandet vid fotofission av ^{238}U under antagande att fissionen beskrivs av penetration genom en symmetrisk och en asymmetrisk barriär. Penetrations-sannolikheterna beräknas med WKB-approximationen. Det asymmetriska bidraget beräknades med barriärparametrar tagna ur litteraturen. Dessa visade sig kunna beskriva energiberoendet hos utbyteskurvan väl. Även den symmetriska fissionen visade sig kunna tolkas med denna modell, under förutsättning att träffytan för symmetrisk fission innehåller kraftiga resonanser.

I arbete III redovisas en undersökning av fissionsfragmentens vinkelfördelning vid fotofission av ^{236}U . Fissionsfragmenten registrerades i tunna folier av polykarbonatplast. Formen på vinkelfördelningen visade sig väl kunna beskrivas av teoretiska uttryck. Med Bohrs kanalmodell kunde läget av nivåer i den till

sadelpunkten deformerade kärnan bestämmas. Enligt denna modell har de lägsta nivåerna i fallet ^{236}U kvanttalen $I, \pi, K = (2, +, 0)$ och $(1, -, 0)$ där I är kärnans spinn, π dess paritet och K spinnprojektionen på kärnans symmetriaxel. De experimentella data upplöstes i komponenter svarande mot fissionsutbytet från dessa nivåer. Med hjälp av modellräkningar liknande dem i arbete II, men med en exakt metod att bestämma penetrationen av fissionsbarriären, erhöles teoretiska utbyteskurvor för fission genom kanalerna $(2, +, 0)$, $(1, -, 0)$ och $(1, -, 1)$. Den första kanalen visade sig väl kunna beskrivas av barriärparametrar för grundtillståndet, erhållna ur andra typer av experiment. Kanalen $(1, -, 0)$ visade sig vara höggradigt degenererad med kanalen $(2, +, 0)$ vid den yttre barriären. Detta är i överensstämmelse med teoretiska uppskattningar. Tolkningen av $(1, -, 1)$ -kanalen förblev en öppen fråga, i hög grad beroende på den osäkra bestämningen av det experimentella utbytet vid låga energier.

Vid det arbete som ligger bakom uppsatserna har av naturliga skäl en viss uppdelning av arbetsuppgifterna skett. Sålunda har jag i arbete II och III endast i mindre grad bidragit vid den teoretiska härledningen och beräkningen av transmissionskoefficienterna för fission och neutronemission.

Till min handledare Professor Bengt Forkman vill jag framföra ett varmt tack för den utomordentliga handledning som kommit mig till del. Hans alltid aktiva intresse för problem i samband med arbetet på denna avhandling har utgjort ett värdefullt stöd.

Till mina medförfattare känner jag stor tacksamhet för trevligt och stimulerande samarbete.

Ett speciellt tack riktas till Professor Olle Wernholm och den övriga personalen i mikrotrongruppen vid Tekniska Högskolan i Stockholm, för den gästfrihet och den oerhörda hjälpsamhet som jag upplevt under mina vistelser vid institutionen.

Slutligen vill jag tacka alla vänner och kollegor inom kärnfotogruppen och inom fysiska institutionen i Lund, som på många sätt hjälpt och inspirerat mig att slutföra detta arbete.

SUMMARY

The present thesis concerns the fission process at low energies, in particular the photofission process in the even-even nuclides $^{236,238}\text{U}$. Fission activities have been induced by means of bremsstrahlung from a microtron, which is a type of electron accelerator.

In the first paper, the valley-to-peak ratio in the fission fragment mass distribution of ^{238}U has been studied as a function of the excitation energy. By using standard radiochemical techniques, isotopes characterizing symmetric and asymmetric fission have been separated. The β -activities of the separated samples have been measured. It is found that the valley-to-peak ratio has a very strong energy dependence in the energy region studied.

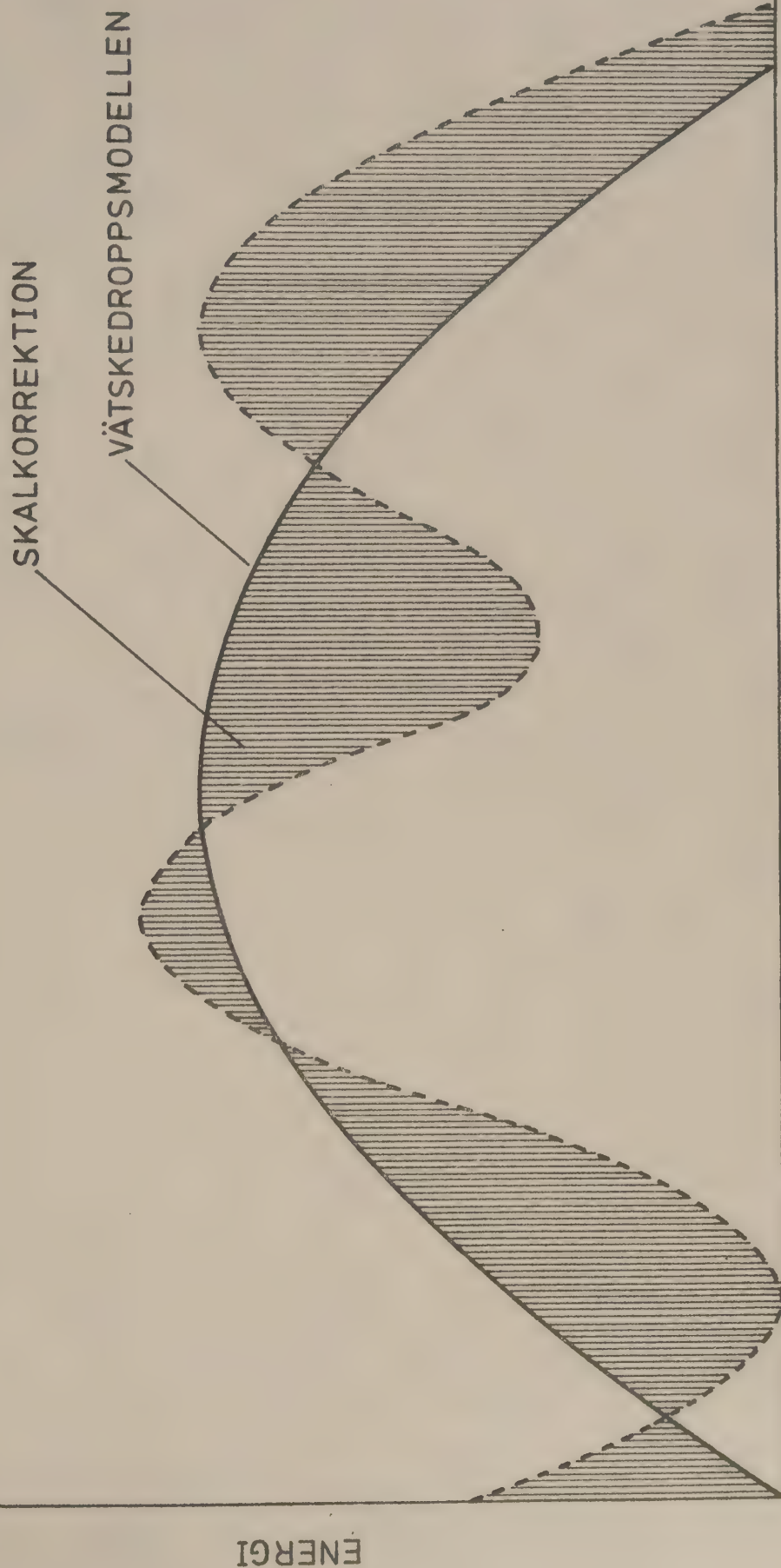
The second paper attempts to give a possible explanation of the experimental results of paper I. By using the assumption that the probabilities for asymmetric and symmetric fission are related to the shapes of the corresponding barrier shapes at the second saddle point, the subbarrier fission cross-sections are calculated for symmetric and asymmetric fission, using the WKB-approximation for the penetration of the double hump barrier. It is found that the experimental data can be explained, at least qualitatively, within the model.

The third paper concerns the angular distribution of fission fragments in the reaction $^{236}\text{U}(\gamma, f)$. The angular distributions and yield of fission fragments are measured, using solid state track detectors, and decomposed into components belonging to different fission channels. The yields of the different channels are compared to theoretical yield curves from model calculations.

It was found that the shapes of the yield curves give information about the shapes of the corresponding fission barriers. In particular, a degeneracy between the channels $1^-,0$ and $2^+,0$ at large deformations is found. This is in agreement with theoretical calculations.

REFERENSER.

- 1) O. Hahn och F. Strassmann, Naturwissenschaften 27 (1939) 89
- 2) N. Bohr och J.A. Wheeler, Phys.Rev. 56 (1939) 426
- 3) O. Haxel, J.H.P. Jensen och H.E. Suess, Phys.Rev. 75 (1949) 1766
- 4) M.G. Mayer, Phys.Rev. 74 (1948) 235
- 5) S.G. Nilsson, Kgl. Danske Videnskap.Selskap, Mat.-Fys.Medd. 29 (1955) no.16.
- 6) W.J. Swiatecki, Phys.Rev. 100 (1955) 937
- 7) S.A.E. Johansson, Nucl.Phys. 12 (1959) 449
- 8) V.M. Strutinski, Nucl.Phys. A95 (1967) 420
- 9) M. Bolsterli, E.O. Fiset, J.R. Nix och J.L. Norton Phys.Rev. C5 (1972) 1050
- 10) P. Möller, Nucl.Phys. A207 (1973) 1
- 11) U. Mosel och H.W. Schmitt, Phys.Rev. C4 (1971) 2185
- 12) H.C. Pauli och T. Ledergerber, Nucl.Phys. A175 (1971) 545
- 13) A. Bohr, Proc.U.N.Int.conf.Peaceful Uses At.Energy (1956), Vol 2, p.151 (New York)
- 14) C.F. Tsang och J.B. Wilhelmy, Nucl.Phys. A184 (1972) 417



FIGUR. Fissionsbarriärens form i olika modeller. Den heldragna parabeln är potentialen i vätskedroppsmodellen (sid. 3). Med Strutinskikorrektionen (sid. 5) erhålles för de tunga aktiniderna den barriär, som anges av den streckade linjen. Områdena mellan kurvorna är skuggade för att framhäva skillnaderna mellan de två potentialerna.

AN EXPERIMENTAL STUDY OF MASS ASYMMETRY IN SUBBARRIER PHOTOFISSION OF ^{238}U

A. ALM and T. KIVIKAS

Department of Physics, University of Lund, Sölvegatan 14, S-223 62 Lund, Sweden

Received 17 May 1973

(Revised 9 July 1973)

Abstract: A measurement of symmetric and asymmetric photofission yields of ^{238}U in the energy region from 4.5 MeV to 6.5 MeV has been performed. As γ -source the bremsstrahlung from a microtron has been used. The fission yields are obtained from β -counting of chemically separated isotopes ^{111}Ag , ^{115}Cd and ^{117}Cd for symmetric fission and ^{139}Ba for asymmetric fission. Some structures in the yield curves at about 5.3 MeV and a pronounced maximum at about 6.0 MeV in the valley-to-peak ratio in the fission fragment yield distribution are observed. It is suggested that these effects are connected with the double-hump fission barrier concept and indicate differences between fission barriers for symmetric and asymmetric fission.

E NUCLEAR FISSION $^{238}\text{U}(\gamma, f)$, $E \approx 4.5\text{--}6.5$ MeV; measured yields ^{111}Ag , ^{115}Cd , ^{117}Cd and ^{139}Ba .

1. Introduction

During the past years great advances in our understanding of the physics connected with the fission process have been made. These advances were initiated by Strutinsky¹⁾ who introduced new theoretical means of calculating the potential energy of a nucleus as a function of its shape. For a nucleus in the actinide region to undergo fission it has to pass over two saddle points, which are separated by a secondary minimum. The minimum occurs because of degeneracies in the single particle energies, especially for deformations where the nucleus is twice as long as it is thick [Nix²⁾]. With this minimum in the fission barrier new experimental discoveries such as spontaneously fissioning isomers³⁻⁸⁾ and broad resonances⁹⁾ in the fission cross sections are naturally explained.

Another important feature of the two-humped fission barrier model is the uniform treatment of the fission-fragment mass distribution. The symmetric and asymmetric mass distributions of the fission fragments are tied together via the first barrier which is common to all mass divisions between the fission fragments. A high second symmetric barrier for symmetric fission and a relatively low asymmetric saddle point as suggested by different theoretical approaches [Möller¹⁰⁾, Mustafa *et al.*¹¹⁾] for ^{238}U , give a hint of an answer to the question of why asymmetric mass division of the fragments dominates the fission process at low excitation energies. One should, however, be aware of the fact that the mass division could be altered during the

passage of the nucleus from the saddle point to scission due to dynamics of the fission process. In the static description there appears to be a correlation between the mass ratios and asymmetric distortions at the saddle point, as has been pointed out by Johansson¹²⁾ and later by Möller¹⁰⁾. Recent calculations by Ledergerber and Pauli¹³⁾ with a model including dynamical effects indicate a similar correlation. These observations give strong support to the hypothesis that asymmetric fission fragment masses are indeed already determined at the saddle point distortions. It should therefore be possible to gain information about the fission barriers from studies of the mass distributions of the fission fragments.

Most early experimental work, to which the theoretical calculations can be compared, has been carried out with various kinds of particle induced fission such as (n, f) [ref. 6)], (d, pf) [ref. 9)] and (α , xnf) [ref. 7)] giving compound nuclei in quite complicated energy and spin states, where in many cases interesting effects are erased. Fission induced by photons is, however, very favourable because in the threshold energy region, photoabsorption takes place only through dipole and quadrupole absorption. For a doubly even nuclide, only 1^- and 2^+ states are populated, giving simple compound nuclei. Thus low-energy photofission, for which experimental data are limited and to some extent inconsistent¹⁴⁻¹⁶⁾, should be an excellent complement to particle induced fission in investigating the barrier effects of the fission process including the fission-channel hypothesis of Bohr¹⁷⁾.

We report in this paper on some new and extended measurements of the yield curves of symmetric and asymmetric photofission of ^{238}U . The measurements were performed in the subbarrier energy region from 4.5 MeV to 6.5 MeV for asymmetric fission and 5.1 MeV to 6.5 MeV for symmetric fission. In this energy region the fission process proceeds via the fission barrier penetration and the yield curves should therefore reflect the characteristics of the fission barriers including possible resonance structures for both symmetric and asymmetric fission. Of special interest in that context is the energy dependence of the valley-to-peak ratio in the mass distribution because this quantity should be very sensitive to differences between symmetric and asymmetric fission.

2. Experimental detail

The asymmetric and symmetric fission yields are obtained by measuring the number of ^{139}Ba nuclei and $^{115,117}\text{Cd}$ and ^{111}Ag nuclei respectively, formed in the fission process. For separation radiochemical methods were used. The isotopes were selected for the following reasons: they are all almost pure β -emitters; they have suitable half-lives; they are representative of the different fission modes; except for Cd they are easy to separate chemically. As the photon source, microtron-produced bremsstrahlung has been used.

2.1. IRRADIATIONS

A microtron is a cyclic accelerator giving a pulsed beam of electrons¹⁸). The microtrons¹⁹) used in this experiment are those at the Royal Institute of Technology in Stockholm and at the Physics Institute at Lund. The former has 11 orbits and gives a maximum kinetic energy of 6.5 MeV to the electrons and the latter has 20 orbits with maximum energy of 11.5 MeV. The energy spread is about 1 % being entirely due to synchrotron oscillations. The average electron currents were approximately 50 μ A and 20 μ A for the two accelerators respectively.

For the orbits used, the relation between the magnetic field strength which defines the electron energy and the current in the magnetic coil was determined by the electron resonance (ESR) method with an accuracy of 0.1 %. The magnet current was measured for 20 min before each irradiation, and showed no variation during this "warming-up period". The cross section of the electron beam is approximately represented by an ellipse with its minor axis of length 6 mm perpendicular to the radius of the plane of the electron orbits, and a major axis of 8 mm.

The irradiations took place in the vacuum tank of the microtron and the sample geometry was chosen in accordance with the electron beam cross section. The bremsstrahlung target was a 1 mm thick, water-cooled platinum disc. The sample consisted of about 20 g of natural UO_2 enclosed in a cylindrical jar with 12 mm radius and 30 mm length, placed behind the bremsstrahlung target. The position of the beam on the target was checked before each irradiation, and viewed during each irradiation with an internal TV set-up.

For the measurement of the low-energy part of the asymmetric fission yield and for that of the symmetric yield, natural uranium was used, and cylinders of metallic uranium, 10 mm thick and with a diameter of 12 mm, were irradiated directly in the microtron tank. The samples were cooled by water. The current of the samples was recorded continuously for activation calculations. The performance of the machines was on the whole very stable during the individual irradiations.

2.2. CHEMICAL SEPARATIONS

For Ba and Ag, it was simple to make radiochemically clean samples, since the separations used are very selective. Also, the nuclides have simple decays to stable nuclides. In the case of Cd, the situation was much more difficult. In the first place, there are no really selective chemical steps for separating Cd. Furthermore, the Cd samples contain three isotopes contributing to their activity, namely $^{115}, ^{117}, ^{118}\text{Cd}$. The genetic relationships of these mass numbers are complicated. Therefore, it is extremely important that the samples have a high radiochemical purity.

The chemical separations were performed in the following way. After irradiation, the sample was dissolved in concentrated HNO_3 and 20 mg each of Ba-, Ag- and Cd-carriers were added. The dissolving procedures took less than 5 min for UO_2 and about 1 h for metallic U. The solution was then evaporated to 15-20 ml volume. From the fission products, Ba was precipitated as $\text{Ba}(\text{NO}_3)_2$ with fuming HNO_3 ,

and Ag was precipitated as AgCl with a few drops of concentrated HCl. After this, the solution was diluted to about 300 ml volume, the acidity was adjusted to 0.5 M and CdS was precipitated with H₂S. The further purification of the β -samples followed standard techniques²⁰⁻²³), and the samples were finally precipitated on a filter paper backing, dried, washed, and mounted for β -counting, 2-4 h after the end of irradiation.

2.3. ACTIVITY MEASUREMENTS

In the part of the experiment when UO₂ was used as target material, the resulting activity was measured with an end-window GM β -counter (Philips 18516) in anti-coincidence with a cosmic ray guard counter (Philips 18518). This arrangement, shielded with lead and iron, gave a background of about 3 cpm. The β -activities of the samples were followed at intervals of 10 or 100 min for several half-lives of the different isotopes.

After each measurement, the background counting ratio was measured, to check the counter for possible contamination. In addition, this measurement also gave a check of the stability of the counter. The background level was found to be stable within the limits of statistics; no contamination was ever found. For the low level measurements (with metallic U-samples), an anticoincidence GM counter²⁴) with a background of about 0.3 cpm was used and the same procedure as above was followed.

2.4. EVALUATION OF DATA

In all our calculations, the genetic relationships of the mass numbers compiled by Davies²⁵) were used. In the ¹¹⁷Cd calculations, the branching ratio to the isomeric state was taken to be 0.1 in accordance with Herrmann²⁶). The numbers of nuclei of the different isotopes in the sample at some reference time were extracted with a least squares fitting. For Ba, the reference time was the end of irradiation, for Ag, it was the time when Ag and its precursor Pd are separated and for Cd the time when Cd and its daughter In are separated. Due to the relatively long separation time for Cd, we have not made any direct measurements of the isotope ¹¹⁸Cd. In fitting the curve, we have made corrections for the existence of a small 49 min activity by assuming that the accumulated ¹¹⁸Cd cross section was equal to that of ¹¹⁷Cd. By omitting the first points of the decay curve, where ¹¹⁸Cd could possibly contribute to the activity, we found that this did not alter our ¹¹⁷Cd data significantly and hence conclude that our assumption was reasonable.

The least squares fitting procedure was also tested in another way. By altering the time during which the activities were recorded, we found that a decay curve had to cover about two half-lives of the decaying isotope in order to give unambiguous results with the calculation method used. For shorter times, the obtained yield depended on the measurement time, probably due to difficulties in the method of resolving the activity from the background. From the recorded beam intensity variations

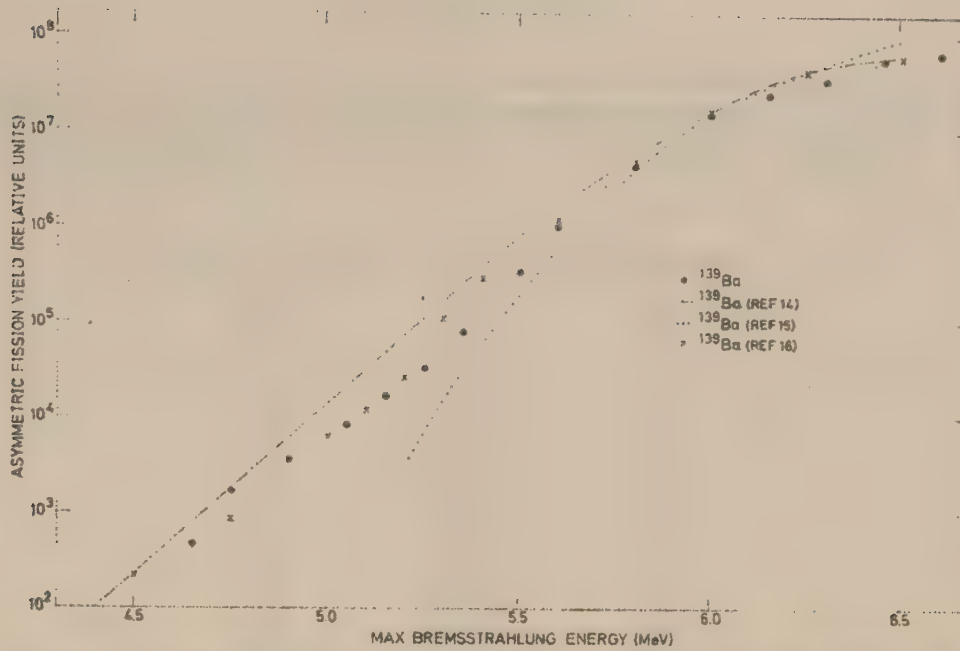


Fig. 1. The yield of the asymmetric fission product ^{139}Ba as a function of maximum bremsstrahlung energy.

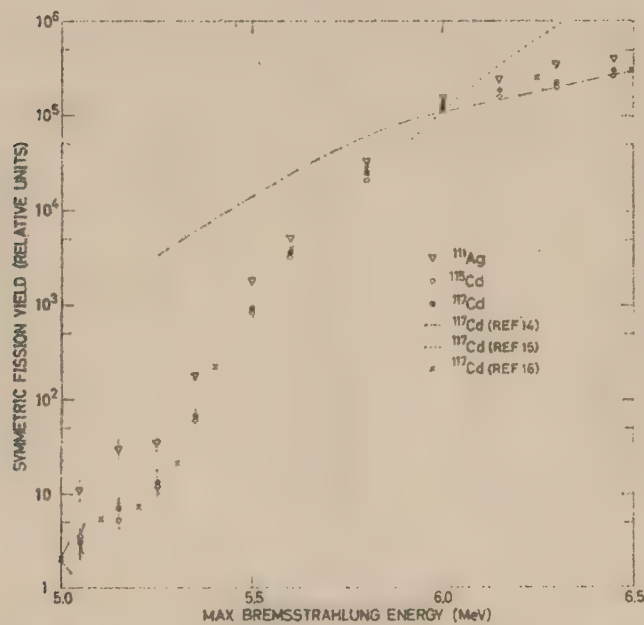


Fig. 2. The yields of the symmetric fission products ^{111}Ag , ^{115}Cd and ^{117}Cd as functions of maximum bremsstrahlung energy.

the activations of the isotopes at the reference times were calculated. The yields of the different nuclei were corrected for the efficiency of the chemical separation and finally normalized to a target weight of 20 g UO_2 or 24 g U, respectively, irradiated with 50 μA for infinite time.

TABLE 1
Yield of fission product ^{139}Ba normalized as stated in the text

Energy (MeV)	^{139}Ba (c/min)	Error (%)
4.65	4.68×10^2	10.7
4.75	1.58×10^3	7.2
4.90	3.76×10^3	5.9
5.05	9.00×10^3	5.0
5.15	1.80×10^4	5.0
5.25	3.58×10^4	5.0
5.35	8.36×10^4	5.0
5.50	3.72×10^5	5.0
5.60	1.06×10^6	5.0
5.80	4.86×10^6	5.0
6.00	1.62×10^7	5.0
6.15	2.74×10^7	5.0
6.30	3.84×10^7	5.0
6.45	6.30×10^7	5.0
6.60	7.20×10^7	5.0

TABLE 2
Yields of fission products ^{111}Ag and $^{115}, ^{117}\text{Cd}$, normalized as stated in the text

Energy (MeV)	^{111}Ag (c/min)	Error (%)	^{115}Cd (c/min)	Error (%)	^{117}Cd (c/min)	Error (%)
5.05	10.5	14.8	3.60	23.2	3.20	25.6
5.15	29.8	14.5	5.30	21.3	7.20	24.5
5.25	34.3	13.3	12.0	17.1	13.5	20.3
5.35	1.75×10^2	11.9	71.0	15.0	61.5	13.4
5.50	1.79×10^3	7.1	8.71×10^2	10.0	9.30×10^2	10.4
5.60	5.06×10^3	5.0	3.35×10^3	7.0	3.60×10^3	6.0
5.80	3.31×10^4	5.0	2.15×10^4	5.0	2.58×10^4	5.3
6.00	1.58×10^5	5.0	1.28×10^5	5.0	1.33×10^5	5.0
6.15	2.96×10^5	5.0	1.66×10^5	5.0	1.90×10^5	5.0
6.30	3.61×10^5	5.0	2.12×10^5	5.0	2.38×10^5	5.0
6.45	4.01×10^5	5.0	2.74×10^5	5.0	3.12×10^5	5.0

3. Experimental results

The photofission yields representing asymmetric and symmetric fission obtained in this experiment are shown in figs. 1 and 2, respectively. They are also listed in tables 1 and 2 respectively. For comparison, data of previous experiments ¹⁴⁻¹⁶) are

also shown in the figures. The different measurements are all normalized at 6.0 MeV. The errors given are composed of the error due to counting statistics and errors due to uncertainties in the chemical separation techniques (the distribution of the final precipitate on the filter-backing etc.). The chemical separation error was estimated to be about 5 % by making several irradiations at the same energy and comparing the normalized counting rates.

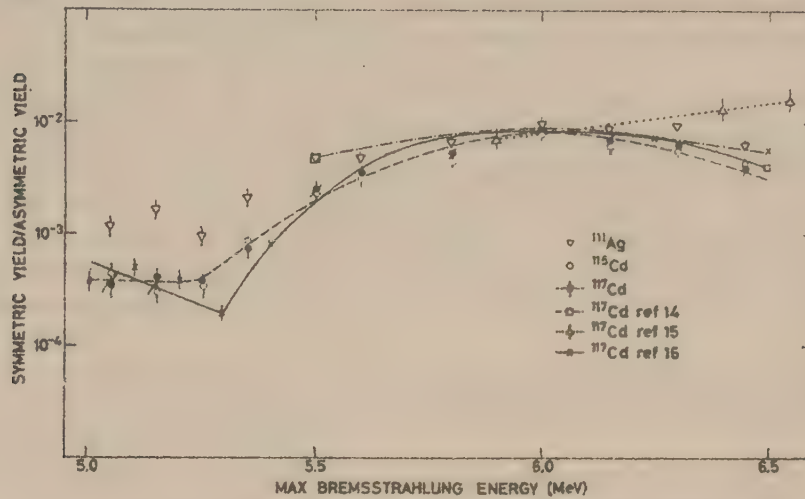


Fig. 3. The ratio between yields of the symmetric fission products ^{111}Ag , ^{115}Cd and ^{117}Cd and the asymmetric fission product ^{139}Ba as functions of maximum bremsstrahlung energy.

TABLE 3
Valley-to-peak ratio in photofission of ^{238}U

Energy (MeV)	$^{111}\text{Ag}/^{139}\text{Ba}$	Error (%)	$^{115}\text{Cd}/^{139}\text{Ba}$	Error (%)	$^{117}\text{Cd}/^{139}\text{Ba}$	Error (%)
5.05	1.17×10^{-3}	19.8	0.43×10^{-3}	28.2	0.36×10^{-3}	30.0
5.15	1.66×10^{-3}	19.5	0.30×10^{-3}	26.3	0.40×10^{-3}	29.5
5.25	0.95×10^{-3}	18.3	0.33×10^{-3}	22.1	0.38×10^{-3}	25.3
5.35	2.09×10^{-3}	16.9	0.85×10^{-3}	20.0	0.74×10^{-3}	18.4
5.50	4.84×10^{-3}	12.1	2.35×10^{-3}	15.0	2.51×10^{-3}	15.4
5.60	4.92×10^{-3}	10.0	3.05×10^{-3}	12.0	3.50×10^{-3}	11.0
5.80	6.75×10^{-3}	10.0	4.39×10^{-3}	10.0	5.30×10^{-3}	10.3
6.00	9.78×10^{-3}	10.0	7.90×10^{-3}	10.0	8.20×10^{-3}	10.0
6.15	9.80×10^{-3}	10.0	6.05×10^{-3}	10.0	6.95×10^{-3}	10.0
6.30	9.40×10^{-3}	10.0	5.10×10^{-3}	10.0	6.20×10^{-3}	10.0
6.45	6.30×10^{-3}	10.0	4.30×10^{-3}	10.0	4.90×10^{-3}	10.0

The statistical error is of importance only in the energy region 5.0–5.3 MeV, where it amounts to 10–30 %.

The asymmetric yield in fig. 1 agrees well with ref. ¹⁶⁾ above 5.5 MeV. Between 5.20 and 5.50 MeV, however, our yield is lower by a factor of about 2, which is out-

side the statistical errors. Below 5.25 MeV, both curves show a linear dependence in a semilogarithmic scale, our results having a somewhat smaller slope. This effect could probably be due to the ^{235}U isotope in the sample which we have not corrected for. Kurchatov *et al.*¹⁶⁾ have made some control runs with depleted ^{238}U and they report a ^{235}U correction which decreases the asymmetric yield by 30 % at 4.8 MeV. Above 5.3 MeV the correction is negligible. Thus the correction is found to be of greatest importance at low energies where the counting statistics are poorest. In accordance with this, we observe the general energy dependence in the low-energy part of our curve to be weaker than the corresponding feature in ref.¹⁶⁾.

For symmetric fission the correction for ^{235}U is unimportant¹⁶⁾ and we also find good agreement between our data and those of ref.¹⁶⁾. Both experiments show a certain structure at 5.3 MeV. The ^{115}Cd yield is systematically higher than the ^{117}Cd yield. The yield of ^{111}Ag lies systematically higher than that of ^{115}Cd and ^{117}Cd by a factor of about 1.3 at 6.5 MeV and about 3.3 at lower energies. This factor reflects the mass distribution function of the fission fragments. Our ^{111}Ag data show good agreement with Kurchatov *et al.*¹⁶⁾. They have however no measurements below 5.3 MeV.

In fig. 3 the energy dependence of the valley-to-peak ratio is shown. The data are given in table 3. As can be seen the maximum at 6 MeV reported by other groups^{14,16)} appears also in the present data.

Our valley-to-peak ratio does not rule out the minimum at 5.3 MeV [ref.¹⁶⁾], although it does not appear pronounced in our data. The correction for ^{235}U would not cause the values to fall outside the error limits indicated in fig. 3. Thus, the tendency for the valley-to-peak ratio to flatten out at lower energies is clearly demonstrated.

4. Discussion

The experimental results are obtained in the form of bremsstrahlung yield curves, i.e.

$$Y(E_{\gamma \max}) = c \int_0^{E_{\gamma \max}} \sigma_f(E_\gamma) N(E_\gamma, E_{\gamma \max}) dE_\gamma,$$

where $\sigma_f(E_\gamma)$ is the fission cross section, $N(E_\gamma, E_{\gamma \max})$ is the bremsstrahlung spectrum for incident electron energy $E_{\gamma \max}$ and c is a constant. Structures in the fission cross section are thus expected to be smeared out, and therefore, for a structure to appear in our experimental data it must be a relatively strong effect.

In a very rough inspection the yield curves obtained show an exponential rise with excitation energy up to about 6 MeV, indicating that the fission process in this energy region can be interpreted as a barrier penetration effect. Due to the very rapid rise of the yield curve however one can expect the fission cross section to have an energy dependence similar to that of the yield. Thus uncertainties of the form of the bremsstrahlung spectrum (thick target corrections) and its effects of damping struc-

tures are depressed because the useful region of the spectrum is limited to a very narrow part at the upper end.

According to the liquid drop model, there should be no structures in that part of the fission cross section which describes the decay of the compound nucleus via barrier penetration, so any structure in the subbarrier energy region should be due to effects in the photoabsorption cross section. Very little is known experimentally about the photoabsorption cross section below 6 MeV. In extrapolated values from data of higher energies no structures are foreseen. However, (t, pf) experiments²⁷⁾ indicate a structure at about the same excitation energy of the compound nucleus as in our photofission yield, which makes it probable that the structure at 5.3 MeV arises from the fission decay of the compound nucleus. Such structures which also have been observed in experiment using monochromatic γ -rays²⁸⁾ can be naturally explained in the double hump barrier concept, which makes allowance for resonances to appear in the subbarrier fission cross section.

The tendency of saturation of the yield curves at about 6 MeV for both asymmetric and symmetric fission can be due either to the passage of a barrier, to the influence of a resonance or to competition from some other decay channel or even possibly due to a combined effect of some of these alternatives. The generally accepted barrier height of about 6 MeV for ^{238}U seems to be in qualitative agreement with our asymmetric yield curve. On the other hand, the symmetric yield is not consistent with a pure penetration through a barrier of about 9 MeV. This height has been proposed recently¹⁰⁾. Neither can the flattening out at about 6 MeV be due to neutron competition exclusively, since the saturation starts at an energy below the neutron emission threshold, $E_n = 6.14$ MeV. Further, the neutron competition should be of the same order of magnitude in both symmetric and asymmetric fission, and therefore not be able to produce such a considerable effect in the valley-to-peak ratio as is observed at 6 MeV. We therefore conclude that this maximum indicates some fundamental difference between the fission channels, including resonant effects, in symmetric and asymmetric fission.

We want to thank professor Bengt Forkman for his never failing interest and support of this experiment, and for many valuable discussions. Our thanks are also due to professor Olle Wernholm at the Royal Institute of Technology in Stockholm for kindly supporting us with experimental facilities. We also thank the staff of the synchrotron laboratory at Lund especially ingenjör Olle Cederholm and the staff of the Royal Institute of Technology in Stockholm. Without the aid of all these people the experiment would never have been completed.

References

- 1) V. M. Strutinsky, Nucl. Phys. **A95** (1967) 420, **A122** (1968) 1
- 2) J. R. Nix, Ann Rev. Nucl. Sci. **22** (1972) 65

- 3) S. M. Polikanov, V. A. Druin, V. A. Karnaukhov, V. L. Mikheev, A. A. Pleve, N. K. Skobolev, V. G. Subbotin, G. M. Ter-Akop'yan and V. A. Fomichev, JETP (Sov. Phys.) **15** (1962) 1016
- 4) S. M. Polikanov and G. Sletten, Nucl. Phys. **A151** (1970) 656
- 5) K. L. Wolf, R. Vandenbosch, P. A. Russo, M. K. Mehta and C. R. Rudy, Phys. Rev. **C1** (1970) 2096
- 6) Yu. P. Gangrsky, K. A. Gavrilov, B. N. Markov, Nguyen Kong Khan and S. M. Polikanov, Sov. J. Nucl. Phys. **10** (1970) 38
- 7) H. C. Britt, S. C. Burnett, B. H. Erkkila, J. E. Lynn and W. E. Stein, Phys. Rev. **C4** (1971) 1444
- 8) V. Metag, R. Repnow, P. von Brentano and J. D. Fox, Z. Phys. **226** (1969) 1
- 9) B. B. Back, J. P. Bondorf, G. A. Otroschenko, J. Pedersen and B. Rasmussen, Nucl. Phys. **A165** (1971) 449
- 10) P. Möller, Nucl. Phys. **A192** (1972) 529.
- 11) M. G. Mustafa, U. Mosel and H. W. Schmitt, Phys. Rev. **C7** (1973) 1519
- 12) S. A. E. Johansson, Nucl. Phys. **22** (1961) 529
- 13) T. Ledergerber and H.-C. Pauli, Nucl. Phys. **A207** (1973) 1
- 14) R. A. Schmitt and R. B. Duffield, Phys. Rev. **105** (1957) 1277
- 15) T. Kivikas and B. Forkman, Nucl. Phys. **64** (1965) 420
- 16) B. V. Kurchatov, V. I. Novgorodtseva, V. A. Pchelin, G. N. Smirenkin, Yu. M. Tsipenyuk and V. M. Shubko, Sov. J. Nucl. Phys. **7** (1968) 326
- 17) A. Bohr, Proc. First Conf. on peaceful uses of atomic energy, paper 911 (Geneva, 1955)
- 18) V. I. Veksler, J. Phys. USSR **9** (1945) 153
- 19) O. Wernholm, Ark. Fys. **26** (1964) 527
- 20) C. D. Coryell and N. Sugarman, Radiochemical studies of the fission products, book 3 (New York, 1957) 1460
- 21) *Ibid.*, p. 1575
- 22) *Ibid.*, p. 1580
- 23) A. C. Wahl and N. A. Bonner, Phys. Rev. **85** (1952) 570
- 24) G. Hultqvist, Swedish Research Institute of National Defence, internal report A 4174-4262 (1960) [in Swedish]
- 25) B. S. J. Davies, Central Electricity Generating Board, Research and Development Department, Berkeley Nuclear Lab, Data for the calculation of gamma radiation spectra from fission products (1971)
- 26) W. Hermann, Radiochimica Acta **3** (1964) 169
- 27) B. B. Back, H. C. Britt, J. D. Garrett and O. Hansen, Phys. Rev. Lett. **28** (1972) 1707
- 28) A. M. Khan and J. W. Knowles, Nucl. Phys. **A179** (1972) 333

Reprint from
"PHYSICS AND CHEMISTRY OF FISSION 1973"
VOL. I

INTERNATIONAL ATOMIC ENERGY AGENCY
VIENNA, 1974

SUBBARRIER PHOTOFISSION OF ^{238}U

A. ALM, T. KIVIKAS, L.J. LINDGREN

Institute of Physics,
University of Lund,
Lund, Sweden

Abstract

SUBBARRIER PHOTOFISSION OF ^{238}U .

Experimental yield curves for symmetric and asymmetric photofission of ^{238}U in the subbarrier energy region are analysed in the framework of a double-hump fission model, where the potential energy is described by three parabolas. In the model, the outer fission barrier is assumed to depend on the type of deformation of the nucleus, thus leading to a coupling between symmetric and asymmetric fission. It is shown that the model reproduces well the experimental data in an energy region where the fission cross-section varies by 5 orders of magnitude. The asymmetric yield can be described by one dominant $1^{-}, 0$ channel with a resonance at 4.6 MeV and one $2^{+}, 0$ channel. The information about the symmetric fission is however very limited, except that a strong resonance at about 5.8 MeV is clearly observed. The maximum at 6 MeV in the valley-to-peak ratio is interpreted as an effect of this resonance in the symmetric yield and not as an effect of neutron competition.

INTRODUCTION

During the past seven years great advances in our understanding of the physics connected with the fission process have been made. These advances have been triggered by new theoretical means of calculating the potential energy of a nucleus as a function of its shape and particle number. According to Strutinski [1], the potential energy for deformation of a nucleus in the actinide region is given by a "double-hump" barrier, i.e. there exists a secondary minimum with a deformation greater than that of the ground state. Approximate one-dimensional calculations with this model explain the occurrence of long-lived fission isomers, as well as certain types of resonances in the fission cross-section. Because of this, the Strutinski theory has been widely accepted in the interpretation of experimental fission data. More detailed calculations with the Strutinski theory, in which account is taken of several deformation coordinates, have yielded three-dimensional surfaces [2] (see fig 1). In these calculations, the deformation coordinates have been divided into two groups, symmetric and asymmetric. This leads to the result that the outer deformation barrier is considerably higher for symmetric fission than for asymmetric. It has also been demonstrated that the inner barrier is common to asymmetric and symmetric fission.

The purpose of this paper is to see to what extent it is possible to find agreement between low-energy fission yield curves for asymmetric and symmetric photofission of ^{238}U and model calculations performed with a somewhat refined model based on these recent theoretical advances.

EXPERIMENTAL PROCEDURE AND RESULTS

The experiment [3] was performed with bremsstrahlung photons produced in a 6.5-MeV microtron. The irradiations took place in the vacuum tank of the microtron. The average current was 20-50 μA and, as target, samples of natural uranium were used. The fission yields have been determined by β -counting of chemically separated isotopes characterizing symmetric and asymmetric fission yields. The activities of $^{115}, ^{117}\text{Cd}$ giving symmetric

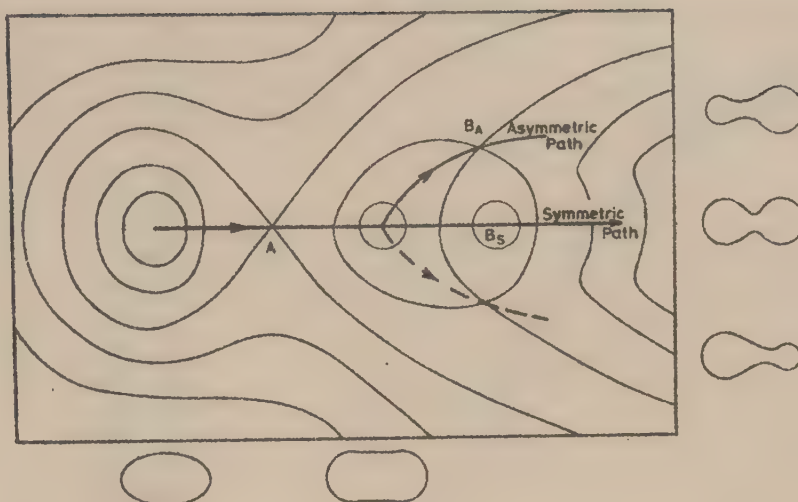


FIG.1. Schematic plot of a deformation energy surface showing the paths for asymmetric and symmetric fission. A indicates the common inner fission barrier, B_A and B_S are the outer fission barrier for asymmetric and symmetric fission respectively.

fission yield were measured for incident electron energies between 5.0 MeV and 6.5 MeV and the activity of ^{139}Ba giving the asymmetric fission yield was measured between 4.6 MeV and 6.5 MeV. The yield curves are shown in figures 2 and 3.

PHOTOFISSION AT LOW ENERGIES

The photofission process at low energies is assumed to be a two-step process. The nucleus absorbs a photon and the compound nucleus thus created decays by fission or by gamma deexcitation or neutron emission

Photo-absorption

It was pointed out by Bohr [4] that the low-energy fission channels for an even-even nucleus should be describable in terms of collective excitations similar to the known collective bands of an actinide nucleus in its ground state deformation. Thus, for ^{238}U we assume that the fission process takes place mainly through 1^- and 2^+ fission channels.

A great drawback when analysing photofission data is the absence of direct, reliable data on the cross-sections for dipole and quadrupole absorption. One has in general to rely mainly on the electro-dynamical estimate $(R/\lambda)^2 \approx 1/50$ for the ratio $\sigma_{\gamma\text{abs}}^{2^+}/\sigma_{\gamma\text{abs}}^{1^-}$, where $2\pi\lambda$ is the wavelength of the photon and R is the nuclear radius. For the energy dependence, the best one can do in the absence of direct data is to extrapolate absorption cross-sections from data at higher energies, keeping the ratio constant. We have simply assumed the approximately exponential energy-dependence given by Khan and Knowles [5] and used the formula

$$\sigma_{\gamma\text{abs}}^{2^+} \approx \left(\frac{R}{\lambda}\right)^2 \sigma_{\gamma\text{abs}}^{1^-}$$

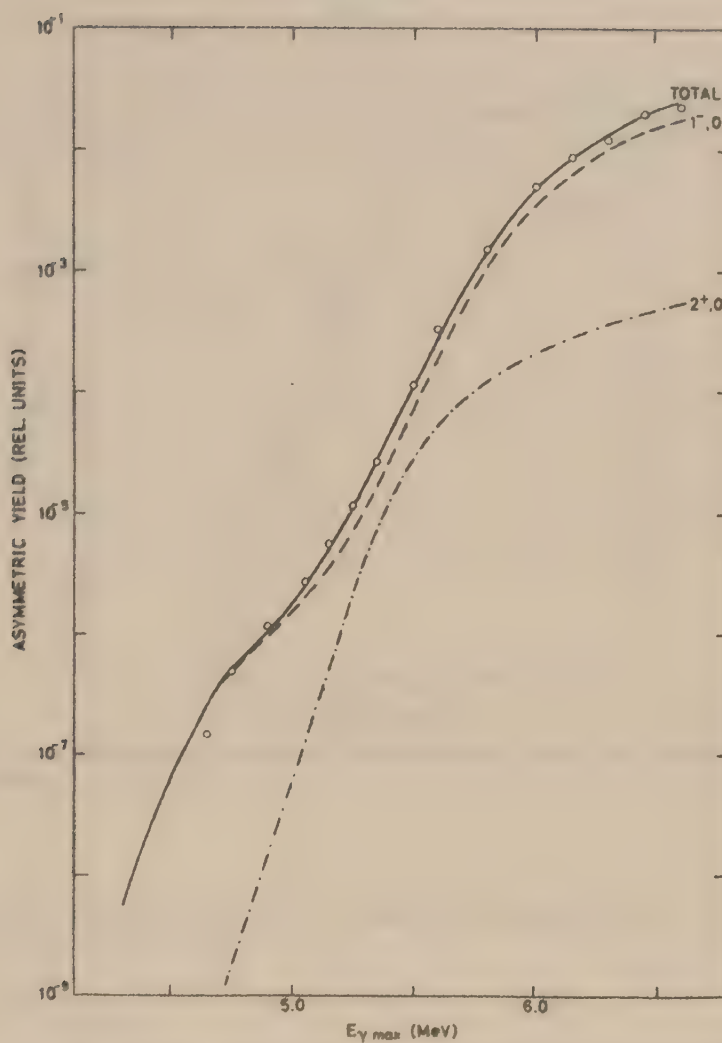


FIG.2. The calculated asymmetric yield curve (solid line) with its channel decomposition compared with the experimental data.

The explicit formula is

$$\sigma_{\gamma\text{abs}}^{1-} = 1.65 \cdot 10^{-3} \exp(-.72 E_{\gamma})$$

Competing decay channels

The possible decay modes of the compound nucleus apart from fission are gamma deexcitation and neutron emission.

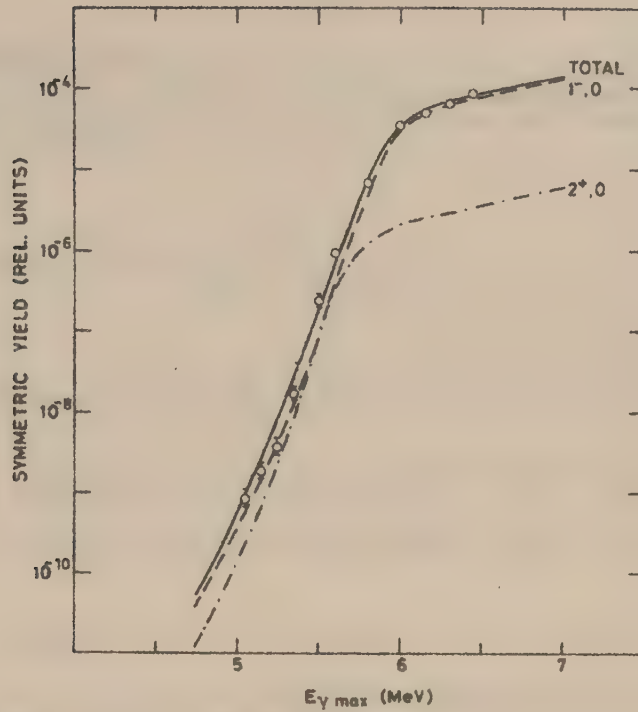


FIG.3. The calculated symmetric yield (solid line) curve with its channel decomposition compared with the experimental data.

The gamma radiation width $\Gamma_Y^{I^\pi}$ is taken from Huizenga and Britt [6]. For the neutron emission width

$$\Gamma_n^{I^\pi} = \frac{D^{I^\pi}}{2\pi} \sum_{\alpha l j} T_{lj}$$

is used, where D^{I^π} is the average distance between levels in the compound nucleus with spin and parity I^π , and T_{lj} is the transmission coefficient for emission of a neutron with angular momentum l and channel spin j . The summation is performed over all combinations of j , l and all available levels α in the ^{237}U nucleus provided that total angular momentum and parity are conserved. The transmission coefficients are taken from Emmerich [7]. At low neutron energies the energy dependence of the transmission coefficient is given by

$$T_{lj} \sim K_{lj} (E_Y - S_n - E_\alpha)^{\frac{2l+1}{2}}$$

Here S_n is the neutron binding energy, E_α is the excitation energy of the ^{237}U nucleus and K_{lj} is a constant.

The fission model

The model is developed on the assumption that the fission process is governed only by barrier transmission. The two-step fission cross-section formula is given by

$$\sigma_f(E_Y) = \sum_I \sigma_{Yabs}^{I\pi} \frac{\sum_K \Gamma_f^{I\pi K}}{\sum_K \Gamma_f^{I\pi K} + \Gamma_Y^{I\pi} + \Gamma_n^{I\pi}}$$

where $I = 1, 2$, π is the parity and the symbols $\Gamma_f^{I\pi K}$, $\Gamma_Y^{I\pi}$ and $\Gamma_n^{I\pi}$ denote, respectively, the widths for fission, γ -deexcitation and neutron emission, the only decay modes possible.

K-mixing is not included in our analysis since our experimental yields can be described mainly by $K = 0$ channels.

According to Bohr-Wheeler [8] the average fission width of a channel is

$$\Gamma^I = \frac{D^I}{2\pi} P^I$$

where D^I is the average distance between compound levels with the same spin and P^I is the barrier penetration.

The expression for barrier penetration [9] is

$$P(E_Y) = \frac{P_A P_B}{4} f$$

where

$$f = \left\{ \cos^2 x \left[\cosh y + \sinh y \frac{P_A + P_B}{4} \right]^2 + \sin^2 x \left[\sinh y + \cosh y \frac{P_A + P_B}{4} \right]^2 \right\}^{-1}$$

P_A and P_B are the penetration of the inner and outer fission barriers respectively and are, according to Hill-Wheeler [10], for a parabolic barrier given by

$$P_A(E_Y) = \left[1 + \exp\left(\frac{2\pi}{\hbar\omega_A} (E_Y - E_A)\right) \right]^{-1}$$

where E_A is the barrier height and $\hbar\omega_A$ is the barrier transparency with

$$\omega_A = \sqrt{\frac{C}{B}}$$

where C is the curvature and B is the mass parameter. In the analysis, a double-hump fission barrier in the form of three conjugate parabolas has been used. $P_A P_B / 4$ is known as P_{min} , the penetration in the absence of resonances. The factor f in the $P(E_Y)$ -expression is due to the fact that

in the double-hump description one expects resonances. If there was no coupling between fission and other degrees of freedom, one would, according to the WKB-approximation, have $f = \{\sin^2 x + \cos^2 x\}^{-1}$, where

$$x(E_Y) = \text{Re } \varphi(E_Y)$$

with

$$\varphi(E_Y) = \frac{\pi}{\hbar\omega_{II}} (E_Y - E_{II} + iW_{II})$$

Here, E_{II} is the energy above the ground state of the secondary minimum and $\hbar\omega_{II}$ is the curvature of the parabola describing the secondary minimum.

We get resonances for the values

$$x(E_Y) = \pi(n + \frac{1}{2}) \quad n = 0, 1, 2, \dots$$

However energy can "leak" from fission to other degrees of freedom and this is taken care of by the complex part

$$y(E_Y) = \text{Im } \varphi(E_Y)$$

in the potential, which results in damping and broadening of the resonances.

The fission cross-section is further split into two parts, one asymmetric $\sigma_f^a(E_Y)$ and one symmetric $\sigma_f^s(E_Y)$, and we get the final expressions

$$\sigma_f^a(s)(E_Y) = \sum_{I^\pi} \sigma_{\gamma\text{abs}}^{I^\pi} \frac{P_f^{I^\pi a}(s)}{\Gamma_f^{I^\pi a} + \Gamma_f^{I^\pi s} + \Gamma_n^{I^\pi} + \Gamma_Y^{I^\pi}} \quad (K = 0)$$

In the model, it is assumed that the inner barrier is common to the symmetric and the asymmetric channels with the same spin and parity and that the fission path is split at the passage of the outer barrier, as indicated in figure 1, in agreement with the calculations of Möller [2].

The asymmetric (symmetric) cross-section can be written

$$\sigma_f^a(s)(E_Y) = \sum_{I^\pi} \sigma_{\gamma\text{abs}}^{I^\pi} \frac{P_f^{I^\pi a}(s) \cdot F^{I^\pi}}{P_f^{I^\pi a} + P_f^{I^\pi s} + P_n^{I^\pi} + P_Y^{I^\pi}}$$

where F^{I^π} is the level density fluctuation factor, which is assumed to be equal to 1.

METHOD OF ANALYSIS

Taking into consideration the fact that the γ -source is a bremsstrahlung beam, the fission yield $Y(E_{Y\text{max}})$ as a function of maximum bremsstrahlung energy $E_{Y\text{max}}$ is given by

$$Y(E_{\gamma\max}) = C' \int_0^{E_{\gamma\max}} \sigma_f(E_\gamma) n(E_\gamma, E_{\gamma\max}) dE_\gamma$$

where $\sigma_f(E_\gamma)$ is the fission cross-section, $n(E_\gamma, E_{\gamma\max})$ is the bremsstrahlung spectrum giving the number of photons of energy E_γ for incident electron energy $E_{\gamma\max}$ and C' is a constant.

Comparison of the experimental results and the theoretical calculations can be based on either the fission cross-sections or the fission yield curves. We have chosen to make use of the latter approach by performing a theoretical model calculation for $\sigma_f(E_\gamma)$, folding in the bremsstrahlung spectrum to obtain the fission yield for comparison with experimental yields. With this method, we can avoid ambiguous solutions when the fission cross-sections are evaluated from the integral equation by a photo-difference method.

For the bremsstrahlung spectrum, the thin target form of Schiff [11] has been used because an accurate thick target spectrum describing a particular experimental set-up is difficult to calculate. Although we in our experiment have definitely had a thick target bremsstrahlung spectrum which should be somewhat softer than the Schiff spectrum, no serious changes in the form of calculated yield curves in the subbarrier energy region were obtained by changing the form of the bremsstrahlung spectrum in order to represent better a thick target spectrum. This effect reflects the very rapid rise of the fission cross-section with excitation energy, thus limiting the useful region of the bremsstrahlung spectrum to a very narrow part at the upper end.

Since our experimental data are insufficient to enable us to make selfconsistent calculations with the model described, it is necessary to make some simplifying assumptions, and to make extensive use of other experimental and theoretical information in order to reduce the number of parameters in the model. We have therefore restricted ourselves to considering only the lowest channels in symmetric and asymmetric fission.

In the case of symmetric fission we have limited ourselves to discussing different possibilities of explaining the fission yield.

Information about the second well is contained only in the resonant part of the yield curve. The location of the resonances on the energy scale is determined by the depth and curvature of the second well. However, it is not possible, from experimental data, to determine the sequential number of a resonance counted from the bottom of the well, since only resonances not too far from the barrier are observed. Therefore, very little is known of the parameters describing the second well. Furthermore, different types of vibrations might have different well parameters. Therefore we have used a theoretical well depth [2] and chosen a curvature that reproduces our resonant effects, assuming that only two resonances are present in the well.

Asymmetric fission yield

According to theory [2], the outer barrier for ^{238}U is asymmetric in shape and is about 6 MeV high. At this deformation the 0^- and 0^+ bands are degenerate in energy [12]. Recent theoretical calculations including non-axial deformations [13] give a value of 5.6 MeV for the inner ground

state barrier. The lowest $2^+,0$ state is expected to be a few hundred keV below the lowest-lying $1^-,0$ state [6].

Measurements of the angular distributions and total fission yields in bremsstrahlung-induced photofission of ^{238}U [14] indicate that the asymmetric fission goes predominantly through a $1^-,0$ channel in the energy region from 5.5 MeV to 6.0 MeV. As a consequence of the above-mentioned facts we expect the fission process in the subbarrier energy region to be described mainly by $K = 0$ channels. With these assumptions our experimental yield curve for asymmetric photofission is analyzed.

Symmetric fission yield

Very little is known about the character of the symmetric fission barrier. There exists no experimental evidence as to which spin and parity values characterize the exit channels. This lack of knowledge leads to considerable ambiguities in the calculations. It is therefore necessary to limit our discussions to different analysis possibilities and to find the minimum number of channels necessary to reproduce the experimental yield curves.

Some facts can be stated:

1. The theory gives an outer symmetric fission barrier of about 9 MeV height, which is unstable against e_3 and e_5 distortions [2]. This greatly diminishes the possibility of a symmetric 1^- channel in the static description.
2. Some experiments indicate resonance structures in the total fission cross-section of ^{238}U at 5.2 MeV and 5.8 MeV.
3. Since the inner barrier is common to asymmetric and symmetric fission, the symmetric yield must be analyzed jointly with the asymmetric yield.

In order to obtain agreement between calculations with a high symmetric barrier, and experimental data, the swift rise of the experimental yield must be reproduced by a strong resonant effect. This is so, since the nonresonant penetrability.

$$P_{\min} \sim \exp \left[\frac{2\pi}{\hbar\omega_A} (E_Y - E_A) \right] \cdot \exp \left[\frac{2\pi}{\hbar\omega_B} (E_Y - E_B) \right]$$

influences the magnitude of the valley-to-peak ratio to a large extent. The higher the barrier, the greater the curvature parameter. The inverse curvature $(\hbar\omega_B)^{-1}$ influences the energy dependence of $P_{\min}$ and a high barrier E_B will give a weak energy dependence. The steep rise in the yield curve must then be due to a strong resonance. The strength has to be so great that the resonance necessarily must be subbarrier, and can therefore hardly be of $2^+,0$ type, since the inner $2^+,0$ barrier is expected to be less than 5.7 MeV [13]. The strength has to be greater, the higher the barrier. In this connection it may be worth pointing out that it is possible to reproduce the low energy part of the symmetric fission yield curve by pure barrier penetration, but it is then necessary to choose an outer barrier height of 5.8 MeV and a curvature of 0.5 MeV. These parameter values are however in violent disagreement with theory. We have therefore in our calculations assumed a high symmetric barrier. The calculations show that it is impossible to explain the symmetric fission yield curve by making use of only one symmetric channel. In this case, only one resonance is possible between 5.3 and 5.8 MeV, and the form of the yield curve

demands a resonance too broad and too strong to be consistent with our calculations. The inclusion of a second resonance channel greatly improves the agreement between the model calculations and the experimental data.

RESULTS OF MODEL CALCULATIONS

The fit of the asymmetric yield was made with the assumption of a dominant $1^{-},0$ channel. The best fit was obtained with $E_A + E_B = 12.15$ MeV. The curve was found to be rather insensitive to small changes in the relative magnitude of the barrier heights. We have used the values $E_A = 6.1$ MeV and $E_B = 6.05$ MeV, as reported also by [14]. In order to reproduce the yield curve in the deep subbarrier region, it was found necessary to include a resonance at 4.6 MeV with a damping of about 0.05, in agreement with [14].

The effective curvature $(\hbar\omega_{eff}) = (1/\hbar\omega_A + 1/\hbar\omega_B)^{-1}$ appeared to have a strong influence on the asymmetric yield curve, and we found the value 0.47 MeV to represent our data best. For simplicity, the curvatures of the single barriers were put equal. The influence of resonances between 5 MeV and 6 MeV is expected to be small, due to considerable damping found in other experiments.

Since the outer $1^{-},0$ and $2^{+},0$ barriers are expected to be degenerate in energy in the asymmetric case, the outer $2^{+},0$ barrier height was found immediately. The inner $2^{+},0$ barrier was assumed to be about 5.6 MeV in agreement with recent theoretical calculations [13]. The curvature was

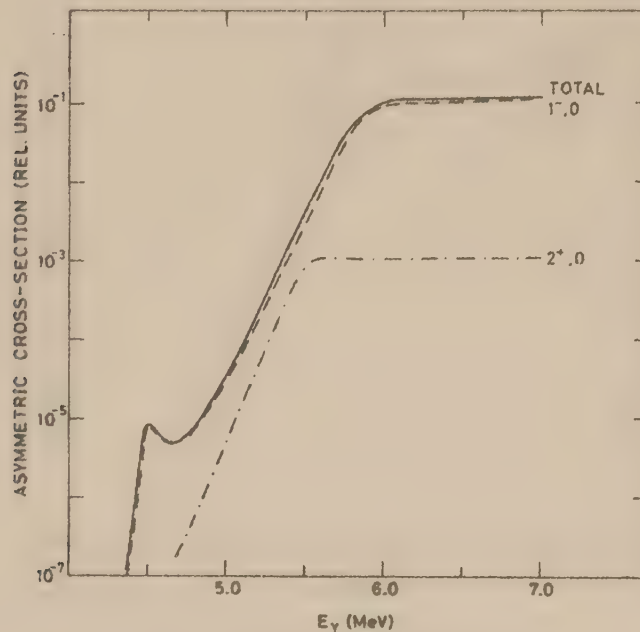


FIG. 4. The asymmetric fission cross-section.

assumed to be about equal to that of the $1^-_{1,0}$ channel, with the same simplifying assumption of equality between inner and outer barriers (i.e. $\hbar\omega_B = \hbar\omega_A$). The quadrupole channel thus determined influenced the yield only below 5.5 MeV.

The results of the calculations and a comparison with the experimental yield curve are shown in fig 2. In fig 4 the cross-section for asymmetric fission is shown.

As mentioned above, information about the character of the symmetric barrier is scarce and the calculations are restricted to some typical cases. The outer barrier is known to be about 9 MeV high and the inner barrier is the same as that of the asymmetric case. Starting from this, the symmetric yield was calculated, using one or two channels.

In the one channel approximation, we considered pure quadrupole and pure dipole fission. According to the calculations by Möller [2], the symmetric barrier is unstable against e_3 and e_5 deformations, which makes it hard to understand the existence of a symmetric $1^-_{1,0}$ channel. The symmetric quadrupole case however is not restricted in that way. The inner barrier height of 5.6 MeV makes it difficult to obtain resonances strong enough to reach agreement between model calculations and experimental data. Our calculations show that a barrier height of at least 6.0 MeV is needed to produce such resonances. Therefore, the question of the character of the symmetric fission exit channel seems to remain open.

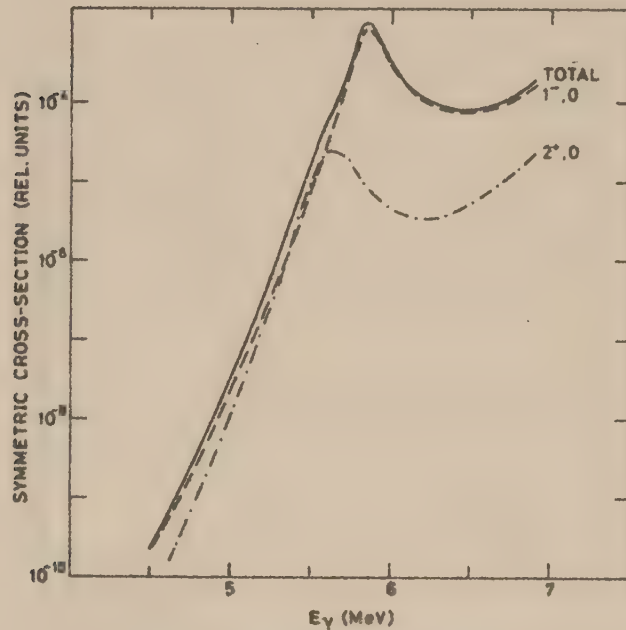


FIG. 5. The symmetric fission cross-section.

TABLE I. BARRIER PARAMETERS FOR THE TWO-HUMPED BARRIER^a

channel I^π, K parameter (MeV)	$1^-, 0$				$2^+, 0$			
	Asym.		Sym.		Asym.	Sym.		
	1	2	3	4		1	2	3
E_A	6.10	6.10	6.10	6.10	5.7		6.0	5.7
$\hbar\omega_A$	1.0	1.0	1.0	1.0	0.9		0.9	0.9
E_B	6.05	8.15	9.2	8.15	6.05		7.5	7.9
$\hbar\omega_B$	0.9	1.5	2.0	1.5	0.9		1.5	1.5
E_{II}	2.90	2.25	2.25	2.25			2.8	2.25
$\hbar\omega_{II}$	3.20	2.40	2.40	2.40			2.0	2.20
V_{II}	-0.05	-0.06	-0.01	-0.06			-0.05	-0.07

^a For the different cases considered in the text, where a detailed definition of the quantities may also be found. In the symmetric analysis, the cases 1 and 2 refer to the pure dipole, 3 refers to a pure quadrupole and 4 to the combined dipole-quadrupole.

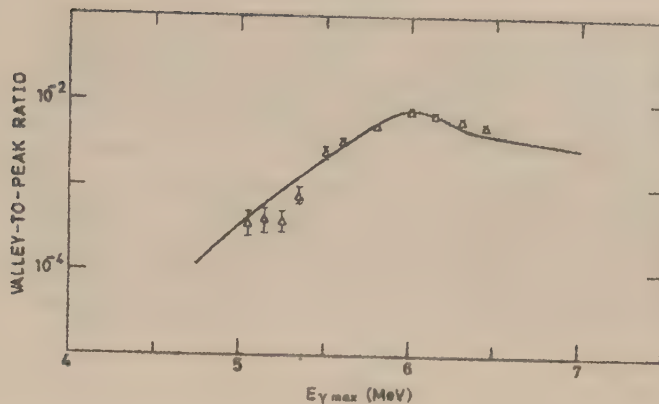


FIG.6. The calculated valley-to-peak ratio for the fission yield curve compared with the experimental data.

In the calculations, a dipole barrier of 9.2 MeV height and curvature of 2.0 MeV gave a reasonable fit to experimental data, using a resonance damping of -0.01 . An equally good fit was however obtained with a lower barrier, 8.2 MeV a smaller curvature, 1.5 MeV and larger damping, -0.05 . To obtain agreement between symmetric quadrupole fission in the one channel approximation and the valley-to-peak ratio, we had to use an inner barrier not less than 6 MeV and an outer barrier of about 7.5 MeV with a curvature of 1.5 MeV. These parameters gave as good an agreement as the dipole case.

However, for the one channel approximation, it was difficult to reproduce properly the behaviour of the symmetric yield between 5.3 MeV and 5.8 MeV. The influence of only one resonance gives too steep an ascent to the curve. Therefore we joined two channels with resonances separated about 0.25 MeV. We have arbitrarily used one $1^-_{1,0}$ and one $2^+_{1,0}$ channel, which gave a much better agreement between model and experiment. It thus seems probable that the symmetric fission process is at least a two channel process, strong resonant effects being associated with it. The results of the calculations and a comparison with the experimental data is shown in fig 3. Fig 5 shows the cross-section of symmetric fission. In table 1 we give the parameter values giving the best fits to experimental yield curves.

Neutron emission does not seem to compete to any significant degree with the resonant effects, and the maximum at 6 MeV, observed in the valley-to-peak ratio as shown in fig 6, seems to originate from the resonant effects of the fission yield together with a local saturation of the fission channel due to the passage of the inner fission barrier.

CONCLUSIONS

On the basis of what has been said, we can draw the following conclusions:

- 1) In the subbarrier region the symmetric and asymmetric yields can be interpreted in terms of penetration of double-hump barriers.

- 2) In order to obtain agreement between the theoretically predicted high symmetric barrier and our experimental data, it is necessary to introduce strong resonances between 5 MeV and 6 MeV. The most significant resonance is the one located at about 5.8 MeV. The maximum at 6 MeV observed in the valley-to-peak ratio is reproduced clearly by the effects of this resonance.
- 3) Neutron competition appears to be unimportant in our model, compared to barrier- and resonance effects. When the neutron threshold is below the fission threshold, the importance of neutron competition is increased. However, in the "penetration" model with a high symmetric barrier, neutron competition can not possibly reproduce the maximum at 6 MeV in the valley-to-peak ratio of ^{238}U .

REFERENCES

- [1] Strutinsky, V.M., Nucl. Phys. A95 (1967) 420.
- [2] Möller, P., Nucl. Phys. A192 (1972) 529.
- [3] Alm, A., Kivikas, T., Nuclear Physics Report LUNP 7302, Lund 1973; Submitted to Nuclear Physics.
- [4] Bohr, A., Int. Conf. peaceful uses atom. Energy (Proc. Conf. Geneva, 1956) 2, UN, New York (1956) 151.
- [5] Khan, A.M., Knowles, J.W., Nucl. Phys. A179 (1972) 333.
- [6] Huizenga, J.R., Britt, H.C., "Threshold Photofission: theory and experiment", Photonuclear Reactions and Applications (Proc. Int. Conf. Asilomar, Cal., 1973), 1, Lawrence Livermore Laboratory, Univ. of California, Livermore (1973) 833.
- [7] Emmerich, W.S., "Fast Neutron Physics, Part II", ch. II A, (Marion, J.B., Fowler, J.L., Eds.) Wiley, New York (1963).
- [8] Bohr, N., Wheeler, J.A., Phys. Rev. 56 (1969) 426.
- [9] Gai, E.V., Ignatiuk, A.V., Rabotnov, N.S., Smirenkin, G.N., Yad. Fiz. 10 (1969) 542; Sov. Journ. Nucl. Phys. 10 (1970) 311 (English Translation).
- [10] Hill, D.L., Wheeler, J.A., Phys. Rev. 89 (1953) 1102.
- [11] Schiff, L.I., Phys. Rev. 83 (1951) 252.
- [12] Brack, M., Damgaard, J., Jensen, A.S., Pauli, H.C., Strutinsky, V.M., Wong, C.Y., Rev. Mod. Phys. 44, (1972) 320; Möller, P., Nilsson, S.G., Phys. Lett. 31B (1970) 283; Nix, J.R., Annu. Rev. Nucl. Sci. 22 (1972) 65.
- [13] Larsson, S.E., Leander, G., Paper IAEA-SM-174/06, these Proceedings, Vol. 1.
- [14] Ignatiuk, A.V., Rabotnov, N.S., Smirenkin, G.N., Soldatov, A.S., Tsipeniuk, Yu. M., Sovj. Phys. JETP 34, 3 (1972) 684.

DISCUSSION

J. R. NIX: Did you find a difference in the mass distribution at a resonance in the cross-section compared to the average mass distribution?

J. R. HUIZENGA (Chairman): Perhaps I could respond to that question. Some 15 years ago Schmitt and Duffield measured the valley-to-peak ratio for photofission of ^{238}U with low-energy bremsstrahlung. This experiment showed a peak in this ratio at $E_{\text{max}} \approx 6$ MeV, indicating that the mass distribution may change at a resonance.

J. R. NIX: The data of Duffield and Schmitt you mention can be interpreted to give the relative penetrabilities of the second asymmetric peak and the first (symmetric) peak in the barrier. This is done by using the WKB expression for the penetrability through a two-peaked barrier. The result is that the second asymmetric peak is more penetrable than the first peak.

T. GOZANI: I would like to make an observation which pertains both to Mr. Back's paper¹ and Mr. Alm's paper². When one compares the available photofission cross-sections for ^{232}Th and ^{238}U with the fission cross-section of the equivalent residual nuclei from charged particle interaction (e. g. $^{230}\text{Th}(t, \text{pf})^{232}\text{Th}$ and $^{238}\text{U}(t, \text{pf})^{238}\text{U}$), one observes the following: the structure in the ^{232}Th cross-section appears in both data at about the same excitation energy, namely 5.6 to 5.8 MeV. It seems however that the structure is more pronounced in the photofission data. On the other hand, the structure in ^{238}U , especially above about 5.3 MeV, which appears clearly in the charged particle data (see Fig. 2, paper IAEA-SM-174/27), hardly appears in the photofission data. Since the experimental uncertainties (mostly energy resolution problems) in the latter are practically the same for both isotopes, I wonder if the difference between the charged particle data and the photofission data reflects the possible excitation of many more angular momentum states in the former as compared to a rather limited number of states (i. e. one or two) which can be excited in this energy range in the photofission process.

B. S. BHANDARI: Mr. Alm, do you take into account any coupling between states in the second well and those in the primary well at corresponding excitation energies?

A. ALM: No. Since our calculations are very phenomenological, we have not dealt with any considerations of that kind.

B. S. BHANDARI: Could this 5.8-MeV resonance be explained in terms of a doorway state?

A. ALM: It possibly could.

H. C. BRITT: I would just point out that there is a strong resonance at 5.8 MeV from $^{238}\text{U}(t, \text{pf})^{238}\text{U}$. This is presumably a resonance in the second well which could be considered a "doorway state".

C. D. BOWMAN: Essentially all photofission measurements have been carried out at energies above 5 MeV and much significant information has been obtained from these measurements. However, I would like to point out that measurements at much lower energies are possible and that interesting results on barrier parameters can be obtained.

¹ BACK, E. B., HANSEN, O., BRITT, H. C., GARRETT, J. D., Paper IAEA-SM-174/27, these Proceedings, Vol. 1.

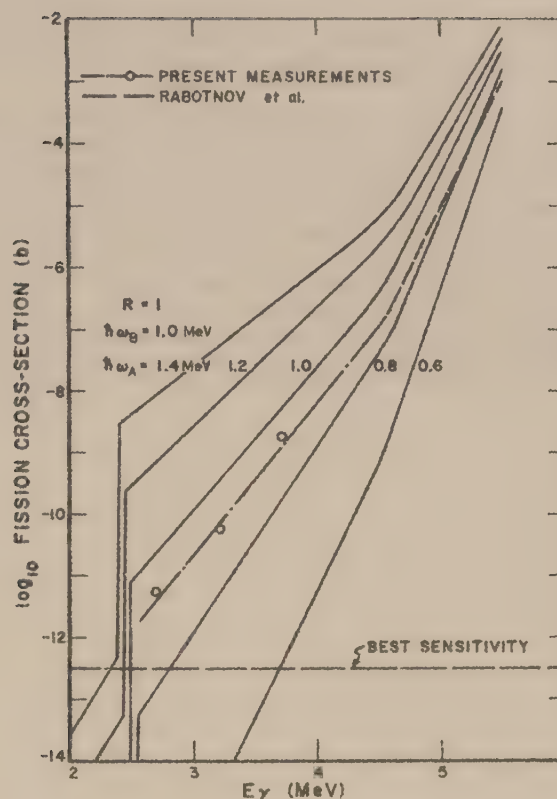


FIG. A. Comparison of subbarrier ^{238}U photofission measurements and calculations.

As the energy of the gamma ray exciting the nucleus decreases, the gamma-ray decay in the second well competes successfully with fission through the outer barrier. As $\Gamma_{\gamma\text{II}}$ becomes greater than Γ_{fII} , the photofission cross-section is dominated by isomeric fission. In this region the photofission cross-section is dependent only on penetration of the inner barrier. The cross-section, therefore, drops less rapidly with energy in this region, and a shelf appears on the photofission cross-section. The cross-section can be analysed for $\hbar\omega_A$ and $\hbar\omega_B$, radiation and fission widths in the second well and other parameters of interest.

Dr. I. Schroder, C. Dick and myself have carried out measurements on ^{238}U down to 2.8-MeV gamma-ray energy and the results are shown in Fig. A. The solid lines represent calculations of the fission cross-section using a method which is an extension of Lynn's very weak coupling formation. The height for both barriers is taken to be 6 MeV and the outer barrier curvature is taken to be $\hbar\omega_B = 1.0$ MeV. The various curves are calculated for different values of $\hbar\omega_A$. The dashed line above 4.5 MeV shows the photofission results of Rabotnov and co-workers. We have measured the cross-section at 3.8, 3.3 and 2.8 MeV using the bremsstrahlung from the 4-MeV electron Van de Graaff accelerator at the National Bureau of Standards.

The dot-dashed line through the points is a straight line fit by eye to our data. The statistical uncertainty in the points is less than $\pm 20\%$. Systematic errors could be as large as a factor of two. A shelf in the cross-section is clearly visible.

We believe that these results confirm our prediction² that measurements in this region are feasible and that such measurements can probably be a useful source of information on fission barrier parameters.

M. S. MOORE: Mr. Bowman, are you suggesting that the dramatic change in valley-to-peak ratio reported by Mr. Alm is related to the change from prompt to isomeric fission as the bremsstrahlung energy is reduced?

C. D. BOWMAN: No. The two effects are entirely unrelated. Alm has shown that low-energy photofission is dominated by asymmetric fission. Since we do not distinguish between symmetric and asymmetric fission, essentially all the fission cross-section and effects we see are concerned with asymmetric fission.

² BOWMAN, C. D., et al., in *Photonuclear Reactions and Applications*, (Proc. Int. Conf. Asilomar, Cal., 1973); Lawrence Livermore Laboratory, University of California, Livermore (1973).

Nuclear Physics Report

LUNP 7506

August 1975

FISSION FRAGMENT ANGULAR DISTRIBUTIONS AND YIELDS OF ^{236}U
IN LOW ENERGY PHOTOFISSION

A. Alm and L.J. Lindgren

Department of Physics, University of Lund, Sölvegatan 14,
S-223 62 LUND, Sweden

FISSION FRAGMENT ANGULAR DISTRIBUTIONS
AND YIELDS OF ^{236}U IN LOW ENERGY
PHOTOFISSION

A Alm and L J Lindgren

Department of Physics University of Lund
Sölvegatan 14, S-223 62 LUND, Sweden

ABSTRACT

A measurement of the angular distribution and yield of fission fragments from photofission of ^{236}U has been performed between 5.2 MeV and 6.4 MeV. As γ -source the bremsstrahlung from a microtron has been used. For the detection of the fission fragments, solid state track detectors were used. The experimental data were analyzed within the framework of the double hump fission barrier model.

NUCLEAR FISSION $^{236}\text{U}(\gamma, f)$, $E \approx 5.2-6.4$ MeV

measured angular distributions and yields.

Enriched target. solid state track detectors.

E

1. INTRODUCTION

One of the most important discoveries in the physics of fission during the last years has been that of the double hump fission barrier by Strutinski¹⁻²⁾. A single-particle shell correction to the liquid drop model results in a modification of the shape of the fission barrier. For nuclei in the actinide region, the fission barrier is found to have the shape of two humps, separated by a secondary minimum at about twice the ground state deformation. This theoretical model has been able to explain several experimental findings, inconsistent with the liquid drop model, such as fission isomerism and the existence of resonance structure in fission cross sections. Particularly, application of this model in the calculation of the deformation potential energy has shown that for the heavy actinides the outer barrier is lowered by several MeV when asymmetric deformations are included and that a saddle point appears at a stable octupole deformation³⁻⁵⁾.

A simple interpretation of the low energy fission process is offered by the Bohr channel hypothesis⁶⁾. For an excitation energy close to the fission threshold, the nucleus, in passing over the saddle point, is assumed to be thermodynamically "cold" since the major part of its energy is bound in the deformation potential energy. Therefore, the excitations available to the nucleus at the saddle point, the "fission channels", should be of the same nature as the low energy excitations of the ground state nucleus. Bohr also pointed out that low energy photofission of even-even nuclei offers an especially simple situation. The spin and parity, I^π , of these nuclei is 0^+ in the ground state. In the energy region appropriate to the fission threshold, photoabsorption takes place mainly

through electric dipole and quadrupole ($E1$ and $E2$) absorption. This yields nuclei with spin and parity 1^- and 2^+ respectively. Close to the barrier, the energy available is insufficient to break a nucleon pair, and the energy levels are due to a few simple collective excitations.

The excited compound nucleus is characterized by the following quantum numbers: the spin and parity I^π , the projection M on a space-fixed axis and the projection K on the symmetry axis of the nucleus. The quantum numbers I and M are conserved throughout the fission process. In the cases where $|M|$ is unique, as in low energy photofission where $|M|=1$, the transition states ("fission channels") are characterized by the quantum numbers I^π , K . The K -projection probably changes when the nucleus passes from the ground state to the saddle⁷⁾, but limited experimental evidence from neutron-induced fission suggests²⁾ that K is a good quantum number in the passage of the nucleus from the transition state to scission. If the quantum numbers I , M , K are conserved, the angular distribution of fission fragments from a transition state with these quantum numbers is uniquely determined, provided only that the fission fragments separate along the nuclear symmetry axis, as is commonly assumed.

The position of the first state $I^\pi K$ above the transition nucleus ground state defines the lowest fission barrier for nuclei excited with the corresponding quantum numbers. By resolving the measured differential fission cross-section into energy-dependent components belonging to specific fission channels it is possible to obtain information about the low energy spectrum of the transition state nucleus, since the shape of

the fission cross-section is sensitive to the height of the fission barrier. Since only a few simple excitations are available in the (γ, f) process of an even-even nucleus at low energies, photofission is a very convenient tool in studying the transition state spectrum of an even-even nucleus.

Fission cross-sections in general have a strong energy dependence at energies close to and below the fission barrier. Therefore strong photon fluxes are required. The only photon flux strong enough to produce measurable fission activities at energies deep below the fission barrier is the bremsstrahlung flux from electron accelerators. This flux has a continuous γ -energy distribution up to the electron kinetic energy. With such a spectrum an integral yield curve is obtained. Despite the problems of unfolding and interpreting such yield curves, it is possible to obtain information about fission barrier characteristics from such measurements. For example, Rabotnov et al⁸⁻¹⁰⁾ have made extensive investigations of the (γ, f) reaction in several even-even actinides using bremsstrahlung produced by a microtron.

In this paper, we report measurements of the (γ, f) process in ²³⁶U. Experimental differential fission yields are decomposed into the yields of the fission channels contributing to the angular distribution. From the energy dependence of these yields information is extracted about the fission channel spectrum.

2. EXPERIMENTAL PROCEDURES

2.1 Irradiations

The experiment was performed with the 6.5 MeV microtron of the Royal Institute of Technology in Stockholm. This electron accelerator has been described in detail elsewhere^{11,12)}.

The electron resonant energy was determined by the number of the orbit and the magnetic field strength. The latter was measured with an accuracy of 0.1% by aid of the electron spin resonance method. The spread about the electron resonance energy was 50-60 keV. With an appropriate choice of the magnetic field strength and the number of the orbit, it was possible to obtain any electron energy between 5 MeV and 6.5 MeV.

The bremsstrahlung converter consisted of a platinum target, 2 mm thick, followed by 1 cm of water and 0.5 mm of copper. This composite target is thick enough to totally absorb the incident electron beam in the energy region 5-6.5 MeV¹³⁾. The net current on the target was integrated automatically and used for normalizing purposes. In this context, we also considered the number of electrons back-scattered from the target. For incident electron energies between 5 MeV and 7 MeV, the number of electrons backscattered from heavy mass targets, such as platinum, is not negligible. The ratio of backward directed electrons to incident electrons is about 0.2¹⁴⁾, with a weak dependence on the incident electron energy E_e . Furthermore, the average energy of the back-scattered electrons is about $0.5 E_e$ ¹⁵⁾, which means that the energy of photons produced by these electrons is not higher than $0.5 E_e$. Due to the strong energy dependence of fission cross-sections at energies below the fission barrier, only photons close to the high energy end of the bremsstrahlung

spectrum can contribute to the fission yields in this experiment. Thus the back-scattered electrons are neither integrated, nor does their radiation interfere with this experiment. In order to define the beam position on the target, an aluminium aperture was placed in front of the bremsstrahlung converter. The electron current hitting this aperture was recorded, and the position of the whole target assembly was varied until this current was at a minimum. For additional control, the aperture was painted with fluorescent ZnS and viewed by an internal television set-up during the irradiations.

The ^{236}U target consisted of 2 mg U_3O_8 mounted as a circular spot with a diameter of 8 mm on a thick aluminium backing. The purity of source was 98.73% ^{236}U with the following specified contaminations: 0.043% ^{234}U , 0.74% ^{235}U and 0.524% ^{238}U . The fission fragments were detected in dielectric track detectors of a polycarbonate plastic foil (the special material used here is available from Bayer AG under the trade name Makrofoil). The thickness of the material was 12 μm , less than the range of fission fragment ($\sim 20 \mu$). Strips of Makrofoil, conveniently cut, were mounted on the circumference of a circular steel cassette, which had a diameter of 40 mm. The cassette was placed 65 mm behind the bremsstrahlung converter with its centre on the extension of the incident electron beam. The angular distributions were measured in angular bins of 15° from -15° to 105° with respect to the photon beam. Each angular bin consisted of a rectangular area 10 mm x 21 mm. The ^{236}U target was mounted in the centre of the cassette with an inclination of 45° relative to the beam. A schematic view of the experimental arrangement is shown in figure 1.

With this experimental configuration, we measured the fission fragment angular distributions and fission yields in steps of 0.1 MeV from 5.2 MeV to 6.4 MeV. The different runs were normalized to each other by means of the total integrated current on the target. The optimal irradiation time depended on the energy and varied from 10 minutes at 6.4 MeV to 10 hours at 5.2 MeV. For each energy, multiple runs were made, and the final experimental distribution was obtained as a sum of corresponding angular bins for the different distributions from separate runs. This method was employed for two reasons: First, the maximum track density in the detectors was about 600 per angular bin (see below). This fact imposed a restriction on the possibility to obtain small statistical errors due to a large number of tracks in a single run. Therefore the summation mentioned above was made. Second, the method permitted a control of the reproducibility of the results from different runs. This was found to be satisfying throughout the experiment.

2.2 Etching and scanning

After the end of the irradiation, the detector strips were etched in 6M KOH at 70°C for one hour. This was done in order to enlarge the holes made by the penetrating fission fragments. After washing and drying, the strips were analyzed by the spark scanning method¹⁶⁾. The etched film was placed as the insulator between a set of plane grounded electrodes and a thin layer of Al at high voltage, evaporated on a Mylar backing. When a high voltage is applied, a spark passes through one of the holes and evaporates a larger hole (≥ 0.1 mm) in the thin Al

electrode. Discharge of a capacitor, parallel to the spark gap, quenches the spark. By proper choice of the capacitor, further discharges are prevented. The spark therefore passes through another hole, jumping from one hole to another until it has passed through all the holes, and the discharges then stops.

The strips were prescanned with a voltage of 900 V in order to enlarge the holes, then scanned with 600 V. This method yielded reproducible results, both absolutely and for the shape of the angular distribution for a certain energy.

When the spark-scanning was completed, the number of holes in the Al electrode were counted in a magnifyer.

The optimal track density in the detectors was found to be 600. The counting error due to the observer was estimated to be 1% of less. For higher track densities, this error increased.

3. EVALUATION OF EXPERIMENTAL RESULTS

The differential yield of fission fragments in photofission of an even-even target is theoretically described by the expression (see 4.2 below).

$$\frac{dY}{d\Omega}(\theta) = a + b \sin^2\theta + \sin^2 2\theta \quad (1)$$

Experimentally, however, the angular resolution is finite. This is indicated in figure 2. Also, since the target has a finite thickness, fragments that leave the target at large angles θ relative to the target normal suffer a greater energy loss than the fragments that leave it close to the normal. Since there is

an energy threshold for penetrating the plastic films used, the detection efficiency for large angles - i.e. in angular bins far from the 45° bin - might be lower than the corresponding value in the 45° bin.

The experimental differential yield is given by

$$N(\psi) = K \cdot \int_{\Omega'} \int_{\Omega''} \frac{dY}{d\Omega}(\theta) n(\psi) d\Omega' d\Omega'' \quad , \quad (2)$$

where $d\Omega'$ is the solid angle defined by the vector $\bar{r}'$ from the bremsstrahlung target to the fission target and $d\Omega''$ is the solid angle defined by the vector $\bar{r}''$ from the fission target to the detector (see figure 2).

In order to study the detection efficiency $n(\psi)$, a special experiment was performed. The angular distribution at 100 MeV bremsstrahlung was measured. In this case, the main contribution to the fission activity comes from the giant dipole resonance region, where the angular distribution is isotropic. Two targets were studied: the ^{236}U target, and a very thin ^{238}U target of the same geometry. The two experimental distributions are compared in figure 3. It is evident that there are thick target effects in the ^{236}U target. The reduced efficiency at small and large angles is probably due to the uneven energy loss of the fragments in the target. For the ^{236}U target, the counting rate falls off rather rapidly at angles larger than 45° relative to the target normal, while for the thin ^{238}U target the counting rate remains constant for all angles considered. Since the only difference between the two targets is their thickness, this effect is attributed to the energy loss of the fragments in the thick target, as mentioned above.

The data for the ^{236}U target, shown in figure 3, have been used to correct for the thick target effects in the source. Using the theorem of the mean, we write

$$N(\psi) = \bar{n} \iint \frac{dY}{d\Omega} (\theta) d\Omega' d\Omega'' \quad , \quad (3)$$

$$M(\psi) = \bar{n} \int d\Omega'' \quad . \quad (4)$$

The $M(\psi)$ function has been normalized to the value 1 in the direction of the target normal (45° bin), on the assumption that the detection efficiency is 100 per cent in this direction.

Thus, we obtain

$$F(\psi) = \frac{N(\psi)}{M(\psi)} = a \frac{\iint d\Omega' d\Omega''}{\int d\Omega''} + b \frac{\iint \sin^2 \theta d\Omega' d\Omega''}{\int d\Omega''} + c \frac{\iint \sin^2 2\theta d\Omega' d\Omega''}{\int d\Omega''} \quad (5)$$

The geometrical expressions, as defined by the integrals above, were calculated by the Monte Carlo method. The results are shown in figure 4. In this calculation we assumed that the electron beam cross-section had no extension, and the integrals were evaluated as fourfold rather than as sixfold, since calculations where these effects were taken into account deviated very little from the original results.

Also, the gamma flux over the target was assumed constant. Calculations of the angular intensity distribution for thick target bremsstrahlung spectra in the energy region considered show that there is only a weak angular dependence, and experiments by Brahme¹⁷⁾ confirm this.

From equation (5) the constants a , b and c were calculated, together with their standard deviations, using the least-squares method¹⁸⁾.

In figure 5-7 we present the experimental results and the curves representing the best fit of the constants a , b and c . From these constants, the quantities b/a and c/b are calculated as functions of energy. The results are given in figures 8 and 9. The constants a , b and c , renormalized by the condition $a + b = 1$ are given in table 1 together with the calculated standard deviations. The quantities b/a and c/b , with errors calculated from the standard deviations of a , b and c , are also listed.

A simple integration of (1) results in

$$Y(E_{\gamma\max}) = K(a + \frac{2}{3} b + \frac{8}{15} c) \quad , \quad (6)$$

where K is a constant, independent of energy.

The total yield curve therefore is the sum of the following yields

$$Y_a(E_{\gamma\max}) = K \cdot a(E_{\gamma\max}) \quad , \quad (7)$$

$$Y_b(E_{\gamma\max}) = K \cdot \frac{2}{3} b(E_{\gamma\max}) \quad , \quad (8)$$

$$Y_c(E_{\gamma\max}) = K \cdot \frac{8}{15} c(E_{\gamma\max}) \quad . \quad (9)$$

The constants a , b and c are obtained directly from the least squares fit. The yields (6) - (9) are shown in figure 10. Table 2 gives the calculated yields and their standard deviations.

4. THEORY

The main assumptions, which underlie the analysis of our experimental data are three. First, it is assumed that the photofission

process at low energies is a two step process. The absorption of a photon by the nucleus, creates a compound nucleus, which decays by fission or gamma-deexcitation (when the energy is below the neutron emission threshold). Second, we assume that the major part of the excitation energy at great deformations is tied up in the deformation energy; that is, the excitation spectrum at the saddle point is similar to the ground state spectrum at low excitations (Bohr's channel hypothesis). Third, the deformation potential energy is presumed to be a double hump barrier.

4.1 The fission cross-section

For an isolated level in the compound nucleus, with spin I and parity π , the fission cross-section can be written as

$$\sigma_f(\lambda, I, \pi) = \sigma_\gamma(\lambda, I, \pi) \cdot \frac{\Gamma_f^{\lambda I \pi}}{\Gamma^{\lambda I \pi}}, \quad (1)$$

where σ_γ is the cross-section for the population of the compound λ (in this case the γ -absorption cross-section), $\Gamma_f^{\lambda I \pi}$ is the width of the exit channel (i.e. the fission channel) and $\Gamma^{\lambda I \pi}$ is the total width of the compound state.

Due to the finite energy resolution in an experiment, it is in most cases impossible to measure the cross-section for a single compound state. The measured quantity is an average value of the strongly fluctuating cross-sections for states in an energy interval ΔE , which contains many compound levels. The total cross-section is given by a sum of the fission cross-sections for the allowed spin and parity values I^π .

Thus, we have

$$\sigma_f(E) = \left\langle \sum_{I\pi} \sigma_f(\lambda I\pi) \right\rangle = \sum_{I\pi} \left\langle \sigma_f(\lambda I\pi) \right\rangle = \sum_{I\pi} \left\langle \sigma_\gamma(\lambda I\pi) \cdot \frac{\Gamma_f^{\lambda I\pi}}{\Gamma^{\lambda I\pi}} \right\rangle. \quad (2)$$

In this context it is usually assumed that the entrance- and exit channels are uncorrelated, which results in

$$\sigma_f(E) = \sum_{I\pi} \left\langle \sigma_{\lambda I\pi}(\lambda I\pi) \right\rangle \left\langle \frac{\Gamma_f^{\lambda I\pi}}{\Gamma^{\lambda I\pi}} \right\rangle = \sum_{I\pi} \sigma_\gamma(E) \cdot \left\langle \frac{\Gamma_f^{\lambda I\pi}}{\Gamma^{\lambda I\pi}} \right\rangle R(I, \pi). \quad (3)$$

The factor $R(I, \pi)$ is the so-called "level width fluctuation correction factor". According to Khan and Knowles¹⁹⁾, it varies between 1.0 and 0.85 for photofission in the energy region between 5 MeV and 8 MeV. In this work we have put it equal to 1.

Using the channel hypothesis, we write

$$\sigma_f(E) = \sum_{I\pi} \sigma_\gamma^{I\pi}(E) \frac{\sum_K \left\langle \frac{\Gamma_f^{I\pi K}}{\Gamma^{I\pi}} \right\rangle}{\left\langle \Gamma^{I\pi} \right\rangle}. \quad (4)$$

The quantity $\left\langle \Gamma^{I\pi} \right\rangle$, the total decay width for spin and parity I^π , is given by

$$\left\langle \Gamma^{I\pi} \right\rangle = \left\langle \Gamma_\gamma^{I\pi} \right\rangle + \sum_K \left\langle \Gamma_f^{I\pi K} \right\rangle = \left\langle \Gamma_\gamma^{I\pi} \right\rangle + \sum_K \left\langle \Gamma_f^{I\pi K} \right\rangle, \quad (5)$$

assuming that the exit channels are statistically uncorrelated.

The average fission width $\left\langle \Gamma_f^{I\pi K} \right\rangle$ can be written

$$\left\langle \Gamma_f^{I\pi K} \right\rangle = \frac{D}{2\pi} T_f^{I\pi K}, \quad (6)$$

where D is the average level spacing of the compound nucleus and $T_f^{I\pi K}$ is the fission barrier transmission coefficient. Thus the fission cross-section can be written as

$$\sigma_f(E) = \sum_{I^\pi} \sigma_\gamma^{I^\pi}(E) \frac{\sum_f T_f^{I^\pi K} \cdot h(K)}{T_\gamma^{I^\pi} + \sum_K T_f^{I^\pi K} \cdot h(K)} \quad (7)$$

The quantity $h(K)$ is 1 when $K=0$, and 2 otherwise. It accounts for the double degeneracy of levels with $K \neq 0$.

The quantity $T_\gamma^{I^\pi}$ is defined by

$$T_\gamma^{I^\pi} = \frac{2\pi}{D} \cdot \langle \Gamma_\gamma^{I^\pi} \rangle \quad (8)$$

4.2 Angular distributions

The relation between the angular distribution $W_K^{I^\pi}(\theta)$ and the differential fission cross-section for a particular channel $I^\pi K$ is given by

$$\frac{d\sigma_f^{I^\pi K}}{d\Omega}(\theta) = \frac{1}{2\pi} W_K^{I^\pi}(\theta) \cdot \sigma_f^{I^\pi K} \quad (9)$$

As was mentioned above, the dominating processes for photo-absorption in the energy region considered are electric dipole- and electric quadrupole absorption. These processes result in compound nuclei with spin and parity 1^- and 2^+ respectively.

Dipole photofission, can occur through a transition level with either $K=0$ or $K=1$ or some combination of these two levels. The corresponding angular distributions are given by

$$\begin{aligned}
 W_0^1(\theta) &= \frac{3}{4} \sin^2 \theta I^{\pi, K=1^-, 0} \quad , \\
 W_1^1(\theta) &= \frac{3}{4} - \frac{3}{8} \sin^2 \theta I^{\pi, K=1^-, 1} \quad .
 \end{aligned}
 \tag{10}$$

In the same way, quadrupole photofission can occur through transition states with $K=0, 1$ or 2 or some combination of these states.

In this case, the angular distributions are given by

$$\begin{aligned}
 W_0^2(\theta) &= \frac{15}{16} \sin^2 2\theta \quad , \\
 W_1^2(\theta) &= \frac{5}{8} (2 - \sin^2 \theta - \sin^2 2\theta) \quad , \\
 W_2^2(\theta) &= \frac{5}{8} (\sin^2 \theta + \frac{1}{4} \sin^2 2\theta) \quad .
 \end{aligned}
 \tag{11}$$

Using (9) we obtain

$$\frac{d\sigma}{d\Omega} f^{1^-}(\theta, E) = \frac{1}{2\pi} \left| \frac{3}{4} \sigma_f^{1^-1} + \frac{3}{4} (\sigma_f^{1^-0} - 0.5 \sigma_f^{1^-1}) \sin^2 \theta \right| \quad , \tag{12}$$

$$\begin{aligned}
 \frac{d\sigma}{d\Omega} f^{2^+}(\theta, E) &= \frac{1}{2\pi} \left| \frac{5}{4} \sigma_f^{2^+, 1} + \frac{5}{8} (\sigma_f^{2^+2} - \sigma_f^{2^+1}) \sin^2 \theta + \right. \\
 &\quad \left. + (\frac{15}{16} \sigma_f^{2^+0} - \frac{5}{8} \sigma_f^{2^+1} + \frac{5}{32} \sigma_f^{2^+2}) \sin^2 2\theta \right| \quad .
 \end{aligned}
 \tag{13}$$

Combining these expression, we obtain

$$\begin{aligned}
 \frac{d\sigma}{d\Omega} f(\theta, E) &= \frac{1}{2\pi} \left| \left(\frac{3}{4} \sigma_f^{1^-1} + \frac{5}{4} \sigma_f^{2^+1} \right) + \left(\frac{3}{4} (\sigma_f^{1^-0} - 0.5 \sigma_f^{1^-1}) + \right. \right. \\
 &\quad \left. \left. + \frac{5}{8} (\sigma_f^{2^+2} - \sigma_f^{2^+1}) \right) \sin^2 \theta + \left(\frac{15}{16} \sigma_f^{2^+0} - \frac{5}{8} \sigma_f^{2^+1} + \right. \right. \\
 &\quad \left. \left. + \frac{5}{32} \sigma_f^{2^+2} \right) \sin^2 2\theta \right| \quad .
 \end{aligned}
 \tag{14}$$

This expression can be simplified considerably. The low-lying levels of the transition state should resemble the

low-lying levels at ground state deformation. Therefore the lowest 2^+ state is supposed to be a pure rotation built on the ground state band. The lowest lying 1^- level is expected to have $K=0$. It is not unreasonable to assume that higher 2^+ states, with $K \neq 0$, can be neglected in comparison with $1^-, 1$ states, even if the corresponding barriers are of comparable magnitudes. This is because the ratio between the absorption cross-sections is $\sigma_{\gamma}^{2^+} / \sigma_{\gamma}^{1^-} \sim 0.02^{20}$.

Therefore, at energies close to and below the barrier we have

$$\frac{d\sigma_f}{d\Omega} = \frac{1}{2\pi} \left| \frac{3}{4} \sigma_f^{1^-1} + \frac{3}{4} (\sigma_f^{1^-0} - 0.5 \sigma_f^{1^-1}) \sin^2 \theta + \frac{15}{16} \sigma_f^{2^+0} \sin^2 2\theta \right|, \quad (15)$$

or

$$\frac{d\sigma}{d\Omega} = a + b \sin^2 \theta + c \sin^2 2\theta, \quad (15^1)$$

$$\sigma_f = \sigma_f^{1^-1} + \sigma_f^{1^-0} + \sigma_f^{2^+0}, \quad (16)$$

or

$$\sigma_f = 4\pi \left(a + \frac{2}{3} b + \frac{8}{15} c \right), \quad (17)$$

where

$$\begin{aligned} a &= \frac{1}{2\pi} \frac{3}{4} \sigma_f^{1^-1}, \\ b &= \frac{1}{2\pi} \frac{3}{4} (\sigma_f^{1^-0} - 0.5 \sigma_f^{1^-1}), \\ c &= \frac{1}{2\pi} \frac{15}{16} \sigma_f^{2^+0}. \end{aligned} \quad (18)$$

The cross-sections for the separate channels can be written in the following useful form:

$$\begin{aligned}\sigma_f^{1-1} &= \frac{a}{a + \frac{2}{3}b + \frac{8}{15}c} \sigma_f , \\ \sigma_f^{1-0} &= \frac{\frac{2}{3}b}{a + \frac{2}{3}b + \frac{8}{15}c} \sigma_f ,\end{aligned}\quad (19)$$

(neglecting the $1^-,1$ component in b)

$$\sigma_f^{2+0} = \frac{\frac{8}{15}c}{a + \frac{2}{3}b + \frac{8}{15}c} \cdot \sigma_f .$$

From this we see that a knowledge of the total fission cross-section and the angular distribution permits a calculation of the cross-section for each separate channel.

5. MODEL CALCULATIONS

The experimental yields were obtained as a function of energy in the form of bremsstrahlung yield curves of the form

$$Y(E_{\max}) = K \int_0^{E_{\max}} \sigma_f(E) N(E, E_{\max}) dE , \quad (20)$$

where the function $N(E, E_{\max})$ represents the bremsstrahlung spectrum. For a direct comparison with experiment we have calculated the yields

$$\begin{aligned}Y_a &= 0.75 Y^{1-1}(E) , \\ Y_b &= 0.5 Y^{1-0}(E) , \\ Y_c &= 0.5 Y^{2+0}(E) .\end{aligned}\quad (21)$$

Instead of solving the integral equation, which imposes numerical difficulties, we chose to perform a model calculation of the fission cross-section folding in a bremsstrahlung spectrum, thus obtaining a calculated yield curve for direct comparison.

As is seen from (7) it is necessary to have knowledge of the photoabsorption cross-section, the barrier penetration coefficient and the "gamma penetration coefficient" in order to calculate the fission cross-sections. To calculate the yield curve, a knowledge of the bremsstrahlung spectrum is necessary. Below, we describe the methods employed for calculating these entities.

5.1 The photoabsorption cross-section

The knowledge of the photoabsorption cross-section at low energies is limited. We have adopted an energy dependence by Axel²¹⁾

$$\sigma_Y^{1-}(E) = 5.2 \left(\frac{E}{7}\right)^3 (0.01A)^{8/3} \text{ mb} , \quad (22)$$

where E is in units of MeV. This form of the cross-section reproduces the limited experimental data in this energy region, within experimental errors. According to Blatt and Weisskopf²²⁾ the electric quadrupole absorption cross-section is approximately equal to

$$\sigma_Y^{2+}(E) = 1.2 E^3 \left(\frac{R}{6} \cdot 10^{-13}\right)^4 \cdot 10^{-3} \text{ mb} , \quad (23)$$

where E is in MeV and R, the nuclear radius, in cm. This formula has been used in the calculations in the absence of more detailed information.

5.2 The barrier penetration coefficient

The barrier penetration coefficient was calculated in a one-dimensional double hump barrier model, assuming a barrier consisting of three smoothly joined parabolas. The penetrabilities are obtained in terms of standard parabolic cylinder functions and their derivatives. To allow for the experimental fact that resonances are lower and broader than predicted by the penetrability, a complex part of the potential was introduced in the second well. This complex potential represents a damping of the fission degree of freedom into other degrees of freedom. The details of the calculation are described in an Appendix.

5.3 Calculation of $T_Y^{I\pi}$

The radiation penetrability $T_Y^{I\pi}$ was calculated within the framework of the statistical theory of nuclear reactions²³⁾:

$$T_Y^{I\pi}(E) = 2\pi \frac{\langle \Gamma_{\lambda\gamma}(E') \rangle}{\langle D(J', \pi E) \rangle} \frac{\langle X(E, a) \rangle}{\langle X(E', a) \rangle} \cdot \frac{\rho(J, E)}{\rho(J', E')} , \quad (24)$$

where $\frac{\langle \Gamma_{\lambda\gamma}(E'J) \rangle}{\langle D(J', \pi E') \rangle}$ is the average radiation width divided by the average level spacing for levels of spin J' at excitation energy E' .

$$X(E, a) = (X^4 - 10X^3 + 45X^2 - 105X + 105) , \quad (25)$$

where $X = 2\sqrt{a(E-\Delta)}$, a and Δ are the level density parameter and correction for pairing in the Fermigas model. $\rho(E, J)$ is the Fermigas level density. All the constants $\langle \Gamma_Y(E') \rangle$, $\langle D(J'E') \rangle$, Δ and a were taken from data for the ^{236}U nucleus by Back et al²⁴⁾.

5.4 Bremsstrahlung spectrum

The great drawback in many photonuclear experiments is the appearance of a bremsstrahlung spectrum. Since the shape and absolute value of the bremsstrahlung spectrum depend on the specific experimental situation, a measurement is called for in every case. In cases where no experimental data are available, approximations must be used. Rabotonov et al⁹⁾ have reproduced the shape of experimental thick target spectra by adding thin target spectra and considering energy loss in the target. The absolute value of the bremsstrahlung cross-section was then determined by normalizing the energy content of the calculated spectrum to experimental intensity measurements. They conclude that the spectrum at its high energy end has a linear shape and thus can be written

$$N(E, E_{\max}) = K(E_{\max})(E - E_{\max}) \quad (26)$$

Some years ago, absolute bremsstrahlung cross-sections were measured by O'Dell et al²⁵⁾ in the region 5 MeV to 20 MeV. These spectra very clearly show the linearity at the high energy end. From these data, the constant $K(E_{\max})$ can be determined. We have used this method to obtain bremsstrahlung spectra in absolute units.

It should be noted that the bremsstrahlung converter used by O'Dell et al was 0.6 mm tungsten while ours was 2 mm platinum. A calculation of energy loss straggling with the theory of Blunck and Leisegang²⁶⁾ shows that the electron energy after traversing the 0.6 mm tungsten target has lost an energy of 1 MeV on the average and that no high energy electrons contribute to the energy distribution. Since the cross-sections

considered are strongly energy dependent, photons of energy less than 1 MeV below the maximum energy are of no importance in this experiment. Therefore we have used the linear spectra determined from the data of O'Dell et al, with due corrections for gamma absorption in the back part of the target.

5.5 Calculation of the bremsstrahlung yield

By using the formulas (7) and (20)-(26) we calculated the yields Y_a , Y_b and Y_c . The integration in (20) was performed using Simpson's rule. The accuracy of the calculation was tested by performing a few test calculations with doubled accuracy. It was found that the accuracy of the calculation was about 1 per cent.

The calculated Y_c curve was normalized to the experimental. By this procedure, the Y_a and Y_b curves are automatically normalized to the experiment. Therefore the order of magnitude of these curves in the experimental units is obtained at once, a fact which facilitates the comparison between the experiment and the model.

6. COMPARISON BETWEEN THEORY AND EXPERIMENT

For a specific fission channel, the shape of the double hump fission barrier is described by seven parameters, namely E_A , $h\omega_A$, E_{II} , $h\omega_{II}$, E_B , $h\omega_B$ and V_0 . (See figure 11.) In addition, the damping in the second well introduces the parameters W_0 and w . Experience with the analysis code, the latter being similar to that used by Back et al²⁴⁾ has shown that the predicted

penetrability exhibits varying degrees of sensitivity these parameters. Some can be extracted in a fairly unique way, while others are left almost undetermined.

Information about the second well parameters E_{II} and $h\omega_{II}$ is contained only in the resonant structure of the penetrability. The depth and shape of the well determine the locations of the resonances, and therefore E_{II} and $h\omega_{II}$ are strongly correlated. In our case, we compared the 2^+0 channel of ^{236}U to the ground state, for which an estimate of the E_{II} value is obtained from isomer excitation functions. For the 1^-0 channel the question is more difficult. The resonances observed in the yield curve appear close to the edge of the well, and it is not known in which sequence they occur, when counted from the bottom. When E_{II} is lifted up by an amount corresponding to the distance between resonances, the shape of the yield curve is hardly affected at all.

The damping parameters W_0 and w mainly influence the height and width of the resonances. Back et al²⁴⁾ have analyzed the fission probabilities of several even-even actinides and extracted the typical values $0 \leq W_0 \leq 150$ keV and $0.10 \leq w \leq 0.15$. The connection between W_0 and w is

$$W(E) = -w(E - E_{II} - \Delta_n - \Delta_p) - W_0, \quad (27)$$

where Δ_n and Δ_p denote the pairing gaps for neutrons and protons respectively. $W(E)$ is the strength of the imaginary potential. It is evident from eq. (27) that these values are not independent. In the absence of detailed information we have used the values $W_0 = 50$ keV and $w = 0.10$.

The value of the quantity $h\omega_B$ can be extracted from systematics of fission isomer lifetimes. In some empirical analyses²⁷⁾, it has been found that $h\omega_A$ is larger than $h\omega_B$ and we have used this as a condition for the barrier. Typical values for the curvature energies are $h\omega_A=1.0$ MeV and $h\omega_B=0.6$ MeV. The searches on these quantities applied in the fitting of our data have been restricted to values in this vicinity. The quantity V_0 appears to have very little influence on the shape of the yield curve. It was taken from experimental data on levels at the ground state deformation.

With these restrictions, the yield curve of the 2^+0 channel was calculated. As a starting point, we used the data obtained by Back et al for the ground state barrier, corrected for the small amount of rotation energy of the 2^+0 state. It appeared that the set of parameters used demanded only a small adjustment to make the shape of the calculated yield curve correspond to our data. This adjustment was within the error limits. A variation of the parameters showed that this was indeed the best fit possible.

For the calculation of the 1^-0 channel, we imposed three constraints on the barrier. The deformations ϵ_1 , ϵ_2 and ϵ_3 (see figure 11) were fixed at the values calculated for the 2^+0 channel.

With all assumptions about the barrier, six constraints are imposed upon nine parameters. Therefore three parameters can be determined independently. We chose the barrier heights E_A , E_{II} and E_B .

The 1^-0 channel was examined in the following way: first the

whole barrier was raised by an energy shift, constant for all deformations. With this shift it was found impossible to reproduce the shape of the yield curve. To obtain the correct energy dependence at the high energy end of the curve it was found necessary to lower the outer barrier and raise the inner. With only a small correction of the well E_{II} a fairly good agreement between experiment and theory was obtained, as shown in figure 10.

The investigation of the Y_a curve presented a serious problem. The small magnitude of the yield makes it evident that the $1^-,1$ barrier must be considerably higher than the $1^-,0$ barrier. In the model used, resonances deep below the barrier have considerable strength, which should give a structure in the corresponding bremsstrahlung yield curve. The absence of such structure would demand a change in the basic assumptions used in our model, since the energy dependence of the strength of the imaginary potential would differ from $1^-,0$ states to $1^-,1$ states. A calculation, based on more degrees of freedom than we have used in the other cases would be meaningless, considering the lack of information about damping of vibrational states. The best fit to the experimental data was obtained by shifting the $1^-,0$ barrier 0.45 MeV, with the second minimum at the same height as for the $1^-,0$ barrier. The resulting curve agreed reasonably with the experimental at high energies, but deviated considerably for energies below 5.5 MeV. Possible reasons for this are discussed below.

The criterion used for testing the goodness of the different fits to experimental values, was a subjective inspection by eye. The sensitivity of the yield curve to a change of the different parameters permitted us to estimate the error limits in the

parameters used. The values of the parameters and their estimated errors are shown in table 3.

7. DISCUSSION

According to the Bohr channel hypothesis⁶⁾ the spectrum of the fission channels should resemble the spectrum of low-lying collective states at the ground state deformation. In this case, the lowest collective state is a rotation built on the ground state with $I^\pi, K = 2^+, 0$ and energy 45 keV, which is close to the ground state. For these reasons, the barrier parameters for the $2^+, 0$ state should resemble the ground state parameters. As was mentioned above, we find close agreement between our parameters and the ground state parameters of Back et al²⁴⁾. The inner and outer barriers are of almost equal heights. This agrees with the known systematics of experimental fission barrier heights, which shows that the barriers for mass numbers around 238 are almost equal in height.

At ground state deformation, the lowest collective state with $I^\pi = 1^-$ is an octupole vibration, with $K=0$. This state has not been identified in the ground state spectrum of ^{236}U , but interpolation between neighbouring nuclides²⁸⁾ suggests that its energy should be about 0.75 MeV higher than the ground state energy. This can be compared with the value for the first barrier, obtained from our calculations, which is 0.85 MeV higher than the corresponding value for the $2^+, 0$ barrier.

For larger deformations, corresponding to the second barrier, the result of our model calculations suggests a near degeneracy of the $1^-, 0$ and $2^+, 0$ channels. Theoretically it has been found⁴⁾

that the second saddle point has a stable octupole deformation, in which case Johansson²⁹⁾ has shown that there should be a near degeneracy of the low-lying $1^-,0$ and $2^+,0$ collective states, and this furnishes an explanation for the result of our calculation. The structure in the Y_b curve at about 5.5 MeV is interpreted as being due to a vibration resonance in the second well of the $1^-,0$ barrier. The model calculations cannot reproduce the strength of the experimental structure, possibly due to limitations of the model. A structure of similar strength has been observed experimentally by Rabotnov et al⁹⁾ in photofission of ^{232}Th .

The calculations of the isotropic yield curve showed that no reasonable assumption about the $1^-,1$ channel, as calculated with our constrained barrier shape, could explain the energy dependence of the experimental Y_a -curve. However, a few points can be made. The large experimental errors are explained by the fact that the isotropic yield is small compared to the other components in the differential yield and therefore difficult to extract with the least-squares method. It also seems that the structure in the Y_a curve reflects the pronounced structure in the Y_b curve. This supports the view that a part of the small isotropic yield may arise from scattering, in the fission source, of other components in the angular distribution.

Rabotnov et al¹⁰⁾ have studied this effect and found that it can be large, especially for small values of the constant a .

Another factor which may affect the $1^-,1$ channel is K mixing. It is possible to account for K mixing in the second well in the model used. Inclusion of this effect shows, however, that it is a rather small effect.

In figure 8 the calculated b/a values are compared with the experimental values. It is seen, that the model reproduces the maximum at about 6 MeV, indicated by the experimental data. The disagreement between theory and experiment at energies below 5.5 MeV can be understood as a consequence of the distortion of the experimental yield at low energies. It would be interesting to determine the Y_a yield in the energy region below 5.5 MeV with better accuracy, in order to test the predictions of the model. To reduce the possible influence of fragment scattering on the results, it would be necessary to use a fission target thinner than the one used in this experiment. This would also reduce the fission yield, implying the necessity of extended radiation times.

The calculated c/b curve is shown in figure 9. It agrees reasonably well with the experiment, and reproduces clearly the experimental maximum at about 5.6 MeV. In the model, the maximum is understood as a consequence of the saturation of the Y_b curve between 5.4 MeV and 5.6 MeV, due to resonant effects.

Finally, we would like to point out that the analysis has been performed in a model, in which the absorption cross-section is assumed to have a smooth energy dependence. Any influence on the shape of the yield curves from structure in the absorption cross-section is therefore excluded in our calculations.

CONCLUSIONS

We find that it is possible to describe the reaction $^{236}\text{U}(\gamma, f)$ in terms of the double hump fission barrier. The angular

distributions of the fission fragments support the Bohr channel hypothesis. From the experimental data we can extract values for the ground state fission barrier heights which are in agreement with the known systematics. For the excited $1^{-},0$ state we find a degeneracy with the $2^{+},0$ barrier at the second saddle point, which is supported by theoretical calculations. The analysis of the isotropic component in the angular distribution remains an open question.

ACKNOWLEDGEMENTS

We want to thank Professor Bengt Forkman for support and for stimulating discussions throughout the course of this experiment. Dr Birger Bo Back is kindly thanked for providing us with experimental data on the $^{234}\text{U}(t, pf)^{236}\text{U}$ reaction. Professor Olle Wernholm and the microtron group at the Royal Institute of Technology in Stockholm are gratefully acknowledged for providing us with experimental facilities, for kind hospitality and for a lot of help during the irradiations.

REFERENCES

- 1) V.M. Strutinski, Nucl. Phys. A95 (1967) 420
- 2) S. Bjørnholm and V.M. Strutinsky, Nucl. Phys. A136 (1969) 1
- 3) H.C. Pauli, T. Ledergerber and M. Brack, B34 (1971) 264
- 4) P. Möller, Nucl. Phys. A192 (1972) 529
- 5) M.G. Mustafa, U. Mosel and H.W. Schmitt, Phys. Rev. C7 (1973) 1519
- 6) A. Bohr, Proc. First Conf. on peaceful uses of atomic energy, paper 911 (Geneva, 1955)
- 7) R. Vandebosch and J.R. Huizenga, "Nuclear Fission" ch VA, Academic Press, New York and London (1973)
- 8) A.S. Soldatov, G.N. Smirenkin, S.P. Kapitza and Yu.M. Tsipenyuk, Phys. Lett. 14 (1965) 217
- 9) N.S. Rabotnov, G.N. Smirenkin, A.S. Soldatov, L.N. Usachev, S.P. Kapitza and Yu.M. Tsipenyuk, Sovj. Journ. Nucl. Phys. 11 (1970) 285
- 10) A.V. Ignatyuk, N.S. Rabotnov, G.N. Smirenkin, A.S. Soldatov and Yu.M. Tsipenyuk, Sovj. Phys. JETP 34 (1972) 684
- 11) V.I. Veksler, J. Phys. USSR 9 (1945) 153
- 12) O. Wernholm, Ark. Fys. 26 (1964) 527
- 13) M.J. Berger and S.M. Selzer, NASA Report NASA-SP-3012 (1964)
- 14) T. Tabata, R. Ito and S. Okaba, Nucl. Meth. 94 (1971) 509
- 15) K.A. Wright and J.G. Trump, Journ. Appl. Phys. 33 (1962) 687
- 16) W.G. Cross and L. Tommasino, Rad. eff. 5 (1970) 85
- 17) A. Brahme (private communication)
- 18) E.M. Pugh and G.H. Winslow, "The analysis of physical measurements", ch. 10, Addison-Wesley, USA (1966)
- 19) A.M. Khan and J.W. Knowles, Nucl. Phys. A179 (1972) 333
- 20) J.R. Huizenga and H.C. Britt, "Threshold Photofission: theory and experiment", Photonuclear Reactions and Applications (Proc. Int. Conf. Asilomar, Cal. 1973) 1 Lawrence Livermore Laboratory, Univ. of California, Livermore (1973) 833

- 21) P. Axel, Phys. Rev. 126 (1962) 671
- 22) J.M. Blatt and V.F. Weisskopf, "Theoretical Nuclear Physics", p. 554, Wiley, New York (1952)
- 23) A.N. Behkami, J. Roberts, W. Loveland and J.R. Huizenga, Phys. Rev. 171 (1968) 1267
- 24) B.B. Back, O. Hansen, H.C. Britt and J.D. Garrett, Phys. Rev. C9 (1974) 1924
- 25) A.A. O'Dell, Jr., C.W. Sandifer, R.B. Knowler and W.D. George, Nucl. Instr. Meth. 61 (1968) 340
- 26) I. Blunck and S. Leisegang, Z. Physik 128 (1950) 580
- 27) K.L. Wolf, Ph.D. thesis, Univ. of Washington, Seattle (1969); S. Bjørnholm, Proc. Robert A. Welch Found. Chem. Res. 13 (1970) 447; as quoted by ref. 7), ch III
- 28) Nuclear Data Sheets B4 (1970) pp. 583-660
- 29) S.A.E. Johansson, Nucl. Phys. 22 (1961) 529
- 30) J.D. Cramer and J.R. Nix, Phys. Rev. C2 (1970) 1048
- 31) J.C.P. Miller, "Handbook of Mathematical Functions" ch. 19, (M. Abramowitz and I.A. Stegun, Eds.) Dover Publications inc., New York (1965)
- 32) D.L. Hill and J.A. Wheeler, Phys. Rev. 89 (1953) 1102

TABLE 1.

Angular distribution parameters a, b and c and their ratios

b/a and c/b, obtained in the reaction $^{236}\text{U}(\gamma, f)$

$E_{\gamma\text{max}}$ (MeV)	a	b	c	b/a	c/b
5.20	0.024 ± 0.005	0.976 ± 0.011	0.155 ± 0.011	41.0^{+}_{-}	0.161^{+}_{-}
5.28	0.029 ± 0.010	0.971 ± 0.025	0.197 ± 0.028	34.0^{+}_{-}	0.202^{+}_{-}
5.35	0.056 ± 0.011	0.944 ± 0.024	0.222 ± 0.027	16.9^{+}_{-}	0.235^{+}_{-}
5.41	0.007 ± 0.011	0.993 ± 0.028	0.354 ± 0.033	$145.^{+}_{-}$	0.316^{+}_{-}
5.46	0.045 ± 0.014	0.954 ± 0.028	0.488 ± 0.034	21.2^{+}_{-}	0.511^{+}_{-}
5.57	0.086 ± 0.017	0.914 ± 0.032	0.430 ± 0.039	10.6^{+}_{-}	0.470^{+}_{-}
5.65	0.086 ± 0.008	0.914 ± 0.015	0.408 ± 0.019	10.7^{+}_{-}	0.446^{+}_{-}
5.75	0.070 ± 0.010	0.930 ± 0.020	0.309 ± 0.023	13.3^{+}_{-}	0.332^{+}_{-}
5.86	0.060 ± 0.008	0.940 ± 0.018	0.241 ± 0.020	15.6^{+}_{-}	0.256^{+}_{-}
5.97	0.054 ± 0.016	0.946 ± 0.034	0.127 ± 0.038	17.5^{+}_{-}	0.134^{+}_{-}
6.07	0.053 ± 0.014	0.948 ± 0.032	0.144 ± 0.035	18.3^{+}_{-}	0.153^{+}_{-}
6.18	0.077 ± 0.012	0.923 ± 0.026	0.097 ± 0.029	11.9^{+}_{-}	0.105^{+}_{-}
6.28	0.074 ± 0.007	0.926 ± 0.014	0.105 ± 0.016	12.6^{+}_{-}	0.113^{+}_{-}
6.39	0.118 ± 0.013	0.882 ± 0.025	0.073 ± 0.028	7.4^{+}_{-}	0.083^{+}_{-}
				10.6^{+}_{-}	0.020^{+}_{-}
				7.1^{+}_{-}	0.018^{+}_{-}
				21.1^{+}_{-}	0.035^{+}_{-}
				9.8^{+}_{-}	0.033^{+}_{-}
				4.7^{+}_{-}	0.036^{+}_{-}
				3.2^{+}_{-}	0.036^{+}_{-}
				∞	0.044^{+}_{-}
				9.2^{+}_{-}	0.042^{+}_{-}
				9.9^{+}_{-}	0.052^{+}_{-}
				5.4^{+}_{-}	0.049^{+}_{-}
				3.0^{+}_{-}	0.061^{+}_{-}
				2.1^{+}_{-}	0.057^{+}_{-}
				3.0^{+}_{-}	0.028^{+}_{-}
				2.1^{+}_{-}	0.027^{+}_{-}
				2.5^{+}_{-}	0.033^{+}_{-}
				1.9^{+}_{-}	0.033^{+}_{-}
				2.8^{+}_{-}	0.027^{+}_{-}
				2.1^{+}_{-}	0.024^{+}_{-}
				8.1^{+}_{-}	0.047^{+}_{-}
				4.4^{+}_{-}	0.044^{+}_{-}
				7.9^{+}_{-}	0.044^{+}_{-}
				4.4^{+}_{-}	0.044^{+}_{-}
				2.7^{+}_{-}	0.035^{+}_{-}
				1.9^{+}_{-}	0.033^{+}_{-}
				1.5^{+}_{-}	0.019^{+}_{-}
				1.2^{+}_{-}	0.019^{+}_{-}
				1.2^{+}_{-}	0.035^{+}_{-}
				0.9^{+}_{-}	0.033^{+}_{-}

TABLE 2.

The bremsstrahlung yields Y_a , Y_b , Y_c and Y ,
as defined in the text.

$E_{\max}$ (MeV)	Y_a (fissions/ μ As)	Y_b (fissions/ μ As)	Y_c (fissions/ μ As)	Y (fissions/ μ As)
5.20	$(1.98 \pm 0.54) \cdot 10^{-3}$	$(5.42 \pm 0.47) \cdot 10^{-2}$	$(6.91 \pm 1.09) \cdot 10^{-3}$	$(6.31 \pm 0.64) \cdot 10^{-2}$
5.28	$(3.38 \pm 1.36) \cdot 10^{-3}$	$(7.66 \pm 0.85) \cdot 10^{-2}$	$(9.33 \pm 1.89) \cdot 10^{-3}$	$(8.86 \pm 1.18) \cdot 10^{-2}$
5.35	$(1.15 \pm 0.31) \cdot 10^{-2}$	$(1.29 \pm 0.12) \cdot 10^{-1}$	$(2.43 \pm 0.47) \cdot 10^{-2}$	$(1.65 \pm 0.20) \cdot 10^{-2}$
5.41	-----	$(1.61 \pm 0.13) \cdot 10^{-1}$	$(4.60 \pm 0.65) \cdot 10^{-2}$	$(2.10 \pm 0.22) \cdot 10^{-1}$
5.46	$(1.14 \pm 0.42) \cdot 10^{-2}$	$(1.61 \pm 0.16) \cdot 10^{-1}$	$(6.59 \pm 0.91) \cdot 10^{-2}$	$(2.39 \pm 0.29) \cdot 10^{-1}$
5.57	$(3.65 \pm 0.98) \cdot 10^{-2}$	$(2.58 \pm 0.27) \cdot 10^{-1}$	$(9.71 \pm 1.58) \cdot 10^{-2}$	$(3.92 \pm 0.53) \cdot 10^{-1}$
5.65	$(7.15 \pm 1.16) \cdot 10^{-2}$	$(5.08 \pm 0.43) \cdot 10^{-1}$	$(1.81 \pm 0.21) \cdot 10^{-1}$	$(7.61 \pm 0.57) \cdot 10^{-1}$
5.75	$(1.14 \pm 0.24) \cdot 10^{-1}$	$(1.01 \pm 0.90) \cdot 10^0$	$(2.67 \pm 0.38) \cdot 10^{-1}$	$(1.39 \pm 0.12) \cdot 10^0$
5.86	$(2.10 \pm 0.44) \cdot 10^{-1}$	$(2.18 \pm 0.20) \cdot 10^0$	$(4.47 \pm 0.71) \cdot 10^{-1}$	$(2.84 \pm 0.32) \cdot 10^0$
5.97	$(3.19 \pm 1.15) \cdot 10^{-1}$	$(3.72 \pm 0.39) \cdot 10^0$	$(3.98 \pm 1.49) \cdot 10^{-1}$	$(4.44 \pm 0.65) \cdot 10^0$
6.07	$(5.78 \pm 1.61) \cdot 10^{-1}$	$(7.06 \pm 0.76) \cdot 10^0$	$(8.62 \pm 2.75) \cdot 10^{-1}$	$(8.50 \pm 1.20) \cdot 10^0$
6.18	$(1.40 \pm 0.32) \cdot 10^0$	$(1.11 \pm 0.10) \cdot 10^1$	$(9.34 \pm 3.41) \cdot 10^{-1}$	$(1.35 \pm 1.70) \cdot 10^1$
6.28	$(1.91 \pm 0.31) \cdot 10^0$	$(1.60 \pm 0.14) \cdot 10^1$	$(1.45 \pm 0.32) \cdot 10^0$	$(1.94 \pm 0.20) \cdot 10^1$
6.39	$(4.57 \pm 0.79) \cdot 10^0$	$(2.27 \pm 0.21) \cdot 10^1$	$(1.50 \pm 0.67) \cdot 10^0$	$(2.87 \pm 0.36) \cdot 10^1$

TABLE 3.

The parameters describing the double hump barrier,
as obtained from the best fits to the experimental data.

Channel Parameter	$2^+, 0$	$1^-, 0$		$1^-, 1$
		case 1	case 2	
E_A (MeV)	5.60 ± 0.20	6.45 ± 0.20	6.42 ± 0.20	6.90
E_{11} (MeV)	2.20	2.20	3.40	2.20
E_B (MeV)	5.55 ± 0.20	5.75 ± 0.20	5.65 ± 0.20	6.20
h_A (MeV)	0.90	0.90	0.89	0.90
h_{11} (MeV)	0.80	0.99	0.73	1.10
h_B (MeV)	0.50	0.49	0.40	0.51
V_0 (MeV)	0.	0.80	0.90	1.25
W_0 (MeV)	0.05	0.05	0.05	0.05
w	0.10	0.10	0.10	0.10

APPENDIX

Calculation of the penetrability through a double hump fission barrier

1. The deformation potential.

In a one-dimensional picture, the potential is approximated by three smoothly joined parabolas. In order to describe the damping of vibration resonances, an imaginary potential of parabolic form is introduced in the second well (see figure 11). The method for the calculation of the penetrability through this barrier is similar to that by Cramer and Nix³⁰⁾, except for the complications arising from the complex part of the potential.

2. Mathematical details.

The type of potential used is shown in figure 11, where the symbols used below also are defined.

$$V(\varepsilon) = \begin{cases} E_A - \frac{1}{2} \mu \omega_A^2 (\varepsilon - \varepsilon_1)^2 & \varepsilon \leq a \\ E_I + \frac{1}{2} \mu \omega_I^2 (\varepsilon - \varepsilon_2)^2 + i \{ W_0 + \frac{1}{2} \mu \omega_{2I}^2 (\varepsilon - \varepsilon_2)^2 \} & a \leq \varepsilon \leq b \\ E_B - \frac{1}{2} \mu \omega_B^2 (\varepsilon - \varepsilon_3)^2 & b \leq \varepsilon \end{cases}$$

The restriction "smoothly joined" yields the following connection formulas:

$$\begin{aligned} \varepsilon_1 &= (2E_A / \mu \omega_A^2)^{1/2} \quad \text{if } V(0) = 0 \\ a &= \varepsilon_1 + [2(E_A - E_I) / \mu \omega_A^2]^{1/2} (1 + \omega_A^2 / \omega_I^2)^{-1/2} \\ \varepsilon_2 &= a + [2(E_A - E_I) / \mu \omega_I^2]^{1/2} (1 + \omega_I^2 / \omega_A^2)^{-1/2} \\ b &= \varepsilon_2 + [2(E_B - E_I) / \mu \omega_B^2]^{1/2} (1 + \omega_B^2 / \omega_I^2)^{-1/2} \\ \varepsilon_3 &= b + [2(E_B - E_I) / \mu \omega_B^2]^{1/2} (1 + \omega_B^2 / \omega_I^2)^{-1/2} \end{aligned}$$

In the following, the indices A, II and B are replaced by 1, 2 and 3 respectively.

The curvature energy of the complex potential is chosen to give

$$\text{Im } V(\epsilon=a) + \text{Im } V(\epsilon=b) = 0$$

With a constant mass parameter , the Schrödinger equation for this particular situation is given by

$$\frac{d^2\psi}{d\epsilon^2} + \frac{2\mu}{\hbar^2} [E - E_i \pm \frac{1}{2}\mu\omega_i^2(\epsilon - \epsilon_i)^2] \psi_i = 0 \quad i=1,2,3$$

In the regions 1, 2 and 3 the following coordinate transformations are made:

$$1. \quad \epsilon \leq a$$

$$u = (2\mu\omega_1/\hbar)^{1/2} (\epsilon - \epsilon_1)$$

$$\alpha_1 = (E_1 - E)/\hbar\omega_1$$

$$2. \quad a \leq \epsilon \leq b$$

$$\tilde{E}_2 = E_2 + iW_0$$

$$\tilde{\omega}_2^2 = \omega_2^2 + i\omega_2^2$$

$$\tilde{\alpha}_2 = (\tilde{E}_2 - E)/\hbar\tilde{\omega}_2$$

$$v = (2\mu\tilde{\omega}_2/\hbar)^{1/2} (\epsilon - \epsilon_2)$$

$$3. \quad b \leq \epsilon$$

$$W = (2\mu\omega_3/\hbar)^{1/2} (\epsilon - \epsilon_3)$$

$$\alpha_3 = (E_3 - E)/\hbar\omega_3$$

With these parameters we obtain the following differential equations:

$$\frac{d^2\psi}{du^2} + (\frac{1}{4}u^2 - \alpha_1)\psi = 0 \quad \epsilon \leq a$$

$$\frac{d^2\psi}{dv^2} + (\frac{1}{4}v^2 + \tilde{\alpha}_2)\psi = 0 \quad a \leq \epsilon \leq b$$

$$\frac{d^2\psi}{dW^2} + (\frac{1}{4}W^2 + \alpha_3)\psi = 0 \quad b \leq \epsilon$$

The solutions to the equations are parabolic cylinder functions, defined in ref. 31. For the regions I and III it is essential to select the proper linear combinations of parabolic cylinder functions, with well-defined directions of phase velocity.

The asymptotic behaviour of the functions at large values of positive and negative ϵ indicates the proper combinations of functions. The directions are marked ($\rightarrow$) and ($\leftarrow$).

In the intermediate region II there is no such restriction, and as solution to the equation we choose a linear combination of y_1 and y_2 , with the symbols used by ref. 30. It must be remembered the variable v is a complex number.

The specific solutions are

$$I) \quad \Psi = A\Psi_1(\rightarrow) + B\Psi_2(\leftarrow)$$

$$\Psi_1 = E^*(\alpha_1, -u) \quad ; \quad \Psi_2 = E(\alpha_1, -u)$$

$$II) \quad \Psi = C\Psi_3 + D\Psi_4 \quad , \quad \Psi_3 = y_1(v) \quad ; \quad \Psi_4 = y_2(v)$$

$$III) \quad \Psi = F\Psi_5 \quad , \quad \Psi_5 = E(\alpha_3, w)$$

The direct transmission is given by

$$P = j_{out} / j_{inc}$$

$$\text{where } j = \hbar/2\mu i \left(\Psi^* \frac{\partial \Psi}{\partial \epsilon} - \Psi \frac{\partial \Psi^*}{\partial \epsilon} \right) =$$

$$= \hbar/2\mu i W(\Psi^*, \Psi)$$

$$j_{inc} = |A|^2 \hbar/2\mu i \left(E(\alpha_1, -u) \frac{\partial E^*}{\partial \epsilon}(\alpha_1, -u) - E^*(\alpha_1, -u) \frac{\partial E}{\partial \epsilon}(\alpha_1, -u) \right)$$

$$\frac{\partial E^*}{\partial \epsilon} = -E'^*(2\mu\omega_1/\hbar)^{1/2}$$

$$\frac{\partial E}{\partial \epsilon} = -E'(2\mu\omega_1/\hbar)^{1/2}$$

$$j_{inc} = -|A|^2 \hbar / 2\mu i (2\mu\omega_1/\hbar)^{1/2} W(E, E^*)$$

$$W(E, E^*) = -2i \quad \text{ref (30)}$$

$$j_{inc} = |A|^2 (\hbar\omega_1/\mu)^{1/2}$$

In the same way $j_{refl} = -|B|^2 (\hbar\omega_1/\mu)^{1/2}$

and $j_{out} = |F|^2 (\hbar\omega_3/\mu)^{1/2}$

The transmission $P = j_{out}/j_{inc} = (\omega_3/\omega_1)^{1/2} |F/A|^2$

The absorbed flux in the second well is given by

$$\Phi_{abs} = j_{inc} - j_{refl} - j_{out}$$

$$\Phi_{abs}/j_{inc} = 1 - R - P \quad \text{where } R = \frac{|B|^2}{|A|^2}$$

The condition of continuity for Ψ and its first derivative gives the relations between the coefficients A, B, C, D and F.

At $E=a$ we have

$$A\psi_1 + B\psi_2 = C\psi_3 + D\psi_4$$

$$A\psi_1' + B\psi_2' = C\psi_3' + D\psi_4'$$

which, after elimination of B, can be written

$$AW^a(1,2) = CW^a(3,2) + DW^a(4,2)$$

The symbol $W^a(i,j)$ means $\psi_i \left(\frac{\partial \psi_j}{\partial E} \right)_{E=a} - \psi_j \left(\frac{\partial \psi_i}{\partial E} \right)_{E=a}$

At $E=b$ we obtain

$$DW^b(4,3) = FW^b(5,3)$$

or $CW^b(3,4) = FW^b(5,4)$

Elimination of C and D gives

$$F/A = \frac{W^a(1,2) W^b(3,4)}{W^b(5,4) W^a(3,2) + W^b(3,5) W^a(4,2)}$$

With the same procedure we also obtain

$$B/R = - \frac{W^b(5,4) W^a(3,1) + W^b(3,5) W^a(4,1)}{W^b(5,4) W^a(3,2) + W^b(3,5) W^a(4,2)}$$

The total penetrability through the barrier is given by

$$T = P + (1 - P - R) \cdot (P_B / P_A + P_B + P')$$

The second part is due to the redistribution of the absorbed flux. The symbols P_A and P_B denote the penetrabilities of the first and second barriers respectively, as given by the exact expression for the penetration of a parabolic barrier³²⁾.

In the figures 13-16 some examples of calculations with the model are shown. The figures show the direct and redistributed parts of the penetrability, and their relative behaviour for various strengths of a parabolic imaginary potential, which has been kept energy-independent in the calculations, in order to show more clearly the effect of its strength on the penetrability.

A very thorough discussion of the model described above, and its application to model calculations of fission transmission coefficients is given by Back et al²⁴⁾.

FIGURE CAPTIONS

- Figure 1 A schematic view of the experimental arrangements.
For details, see the text.
- Figure 2 A schematic view of the experimental geometry.
A photon is generated in the point $(0,0,0)$ of the xyz-system. The photon hits the fission foil at the point $(x',y',0)$ of the $x'y'z'$ -system. The fission fragment is recorded at the point $(x'',y'',0)$ of the $x''y''z''$ -system. The angles (θ_1, φ_1) and (θ_2, φ_2) are chosen from random number distributions for use in the Monte Carlo calculations mentioned in the text. The vector $\vec{r}_d$ is the vector between the centre of the fission foil and the centre of the detector in the θ_d direction.
- Figure 3 The experimental angular distributions at 100 MeV bremsstrahlung for a thin ^{238}U target (∇) and for the ^{236}U target used in the experiment ($\circ$).
- Figure 4 The calculated integral constants, described in the text, normalized to the conditions

$$\int_{\Omega''=2\pi} d\Omega' d\Omega'' = 2\pi \Omega'$$

$$\int_{\Omega''=2\pi} \sin^2 \theta d\Omega' d\Omega'' = \frac{4}{3} \pi \Omega'$$

$$\int_{\Omega''=2\pi} \sin^2 2\theta d\Omega' d\Omega'' = \frac{16}{15} \pi \Omega'$$

Figure 5 The experimental angular distributions for energies 5.2 MeV-5.57 MeV (ϕ), normalized to the value in the 90° direction. The dashed curves represent the components of the angular distribution, as given by the best fit of the constants a , b and c . The solid curve is the sum of these components. Actually, both the experimental values and the calculated components represent histograms, the width of each angular bin being 15° . For greater clarity, the curves are drawn.

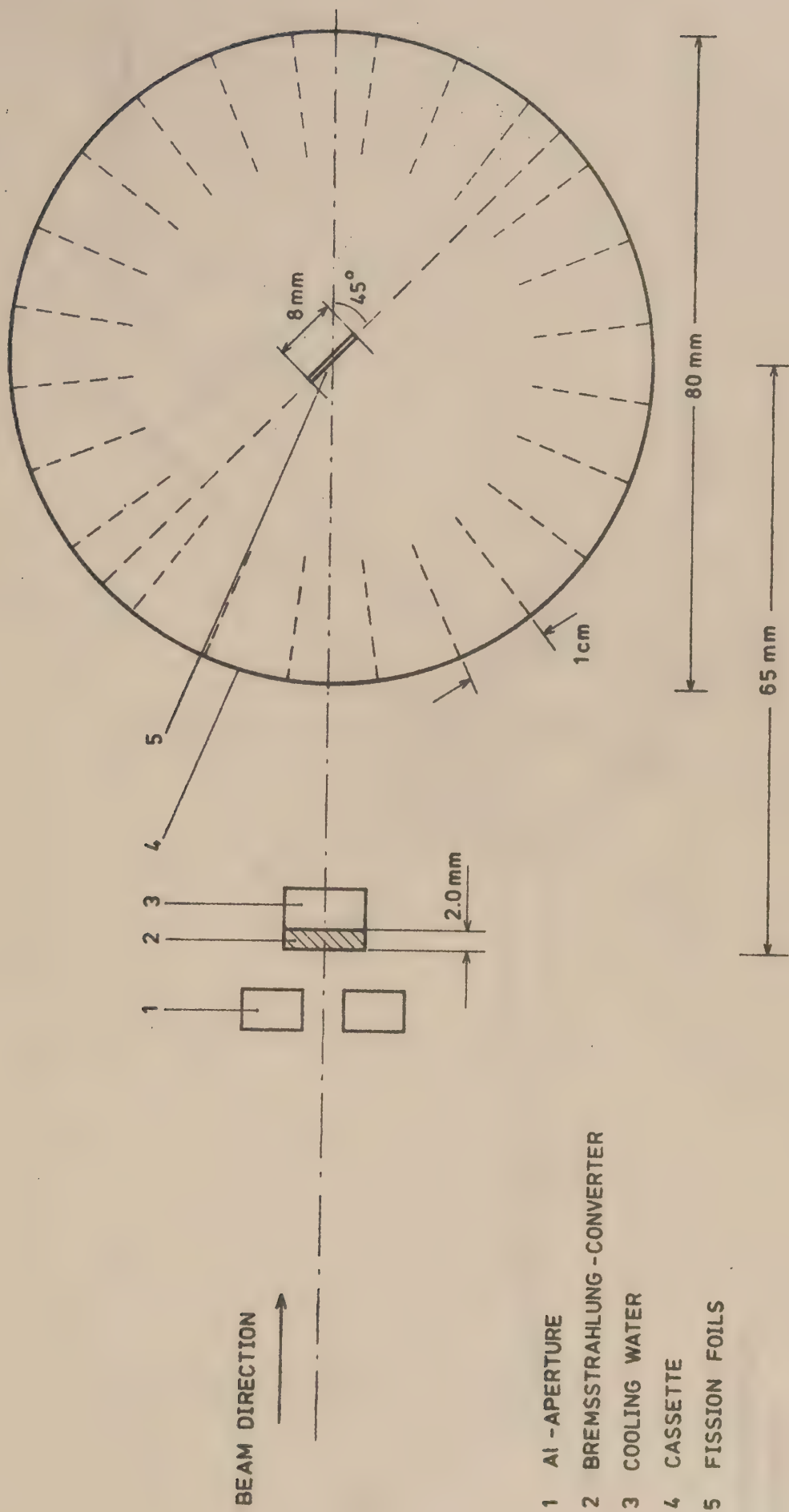
Figure 6 Same as figure 5, for 5.65 MeV-6.18 MeV.

Figure 7 Same as figure 5, for 6.28 MeV-6.39 MeV.

Figure 8 The experimental b/a ratio for ^{236}U (ϕ). The solid curve (—) represents the theoretical b/a ratio, as obtained from the best fits to the Y_a and Y_b curves obtained in the model calculations described in the text. For comparison, experimental b/a values for ^{238}U (ϕ) and ^{232}Th (ψ) from Rabotnov et al.⁹ are shown. In these cases the dashed (---) and dash-dotted (-.-.-) curves are drawn only to guide the eye.

Figure 9 Same as figure 8, for the ratio c/b .

- Figure 10 The yields Y (1), Y_a (3), Y_b (2) and Y_c (4), calculated as described in the text. The curves represent the best fits to the experimental points.
- Figure 11 The shape of the double hump barrier used in the model calculations. In the figure, the parameters describing the barrier are defined.
- Figure 12 The shape of the double hump barrier obtained from the model calculations. The solid line indicates the barrier obtained for the $2^+, 0$ state. The shadowed band describes the $1^-, 0$ barrier, the width of it indicating the error limits.
- Figure 13 The penetrabilities for the direct flux (---) and the redistributed flux (—) in the penetration of the double hump barrier, calculated with $W_0=0.01$, $E_A=6.0$ MeV, $E_{II}=2.5$ MeV, $E_B=5.5$ MeV, $w=0$, $h\omega_A=0.9$ MeV, $h\omega_{II}=0.8$ MeV and $h\omega_B=0.5$ MeV.
- Figure 14 Same as figure 13, with $W_0=0.1$.
- Figure 15 Same as figure 13, with $W_0=0.5$.
- Figure 16 The total penetrability, obtained as a sum of the direct and the redistributed parts. The dotted curve (.....) is for $W_0=0.01$, the solid curve is for $W_0=0.1$, and the dash-dotted curve is for $W_0=0.5$.



- 1 Al - APERTURE
- 2 BREMSSTRAHLUNG - CONVERTER
- 3 COOLING WATER
- 4 CASSETTE
- 5 FISSION FOILS

Fig. 1

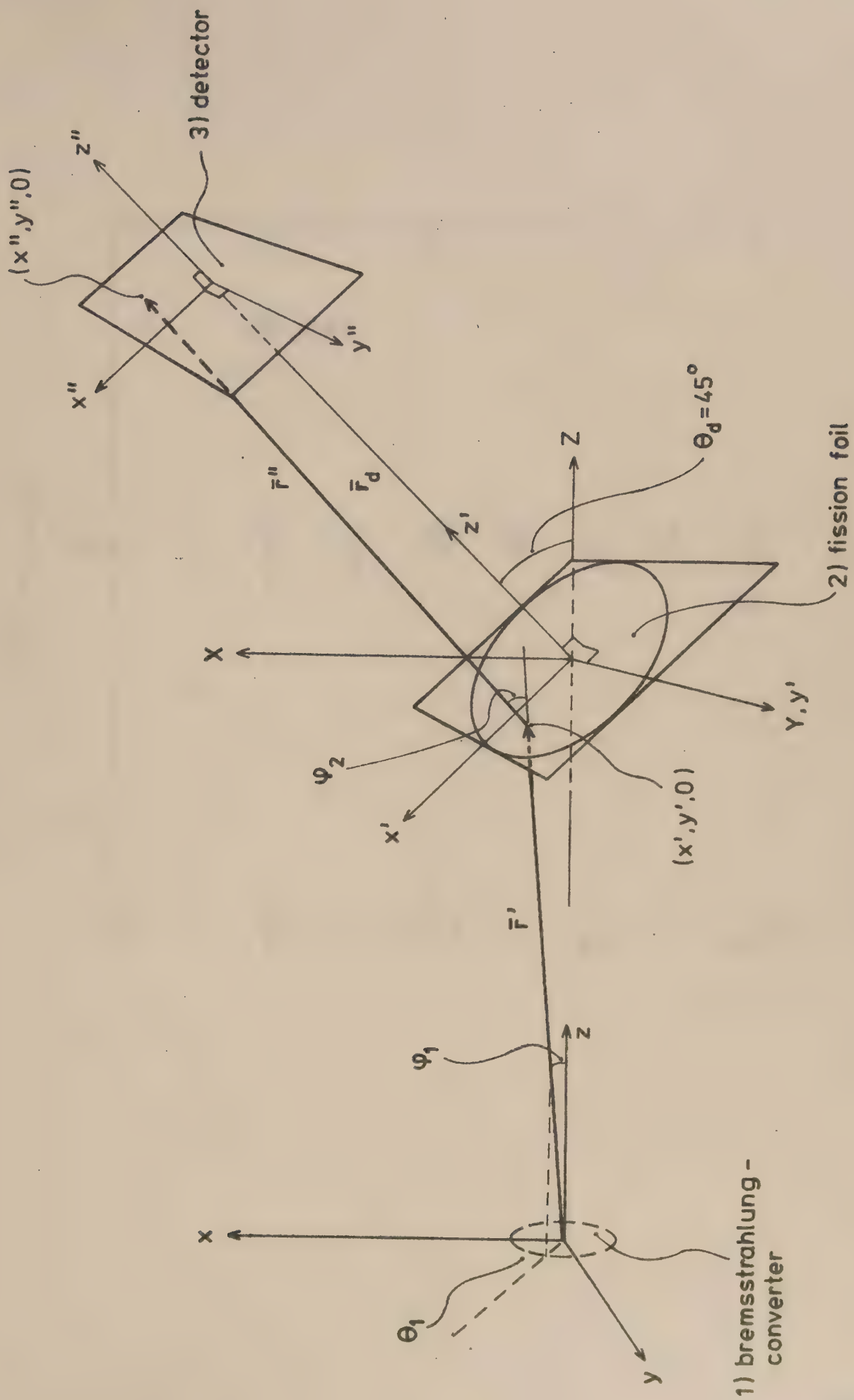


Fig. 2

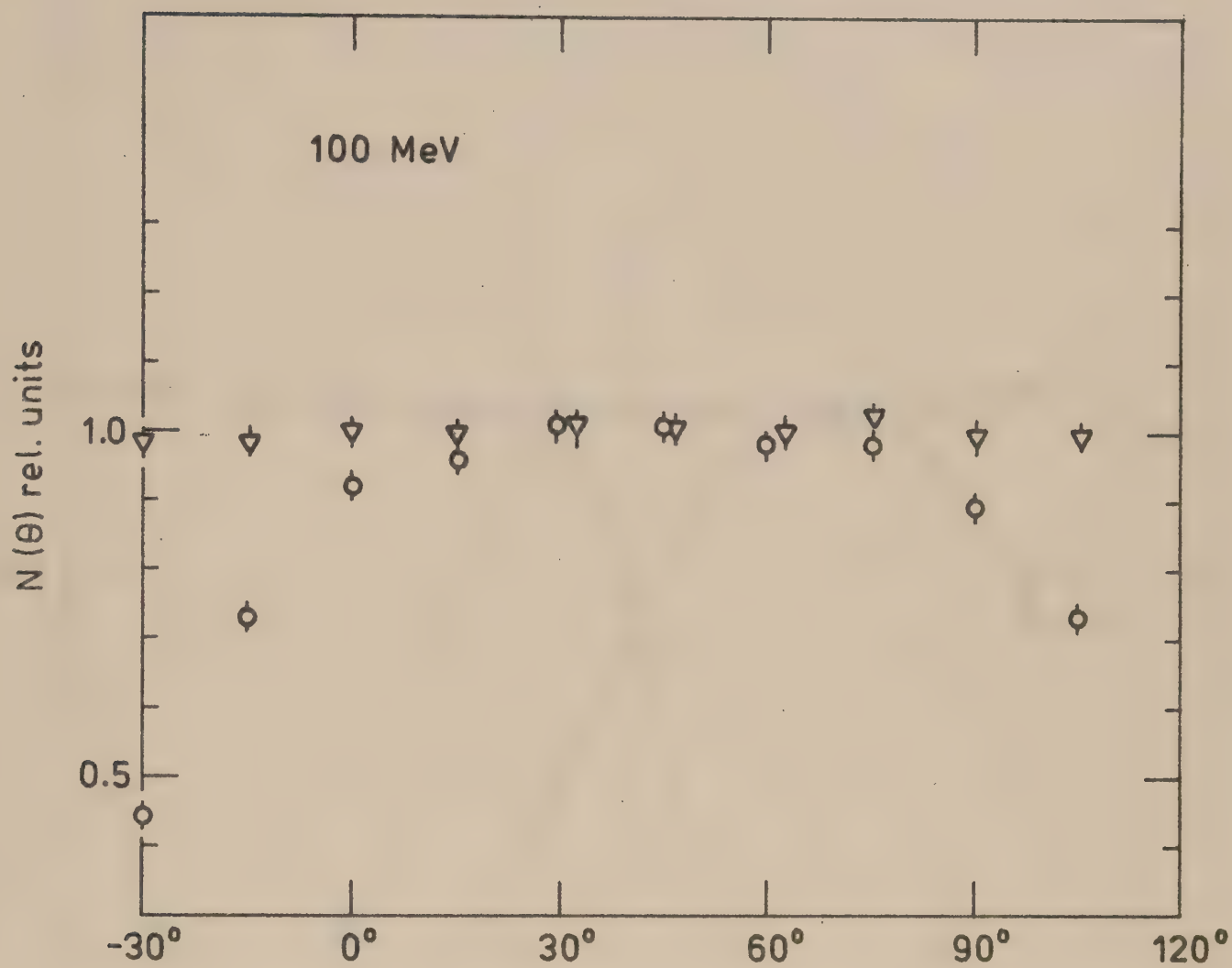


Fig. 3

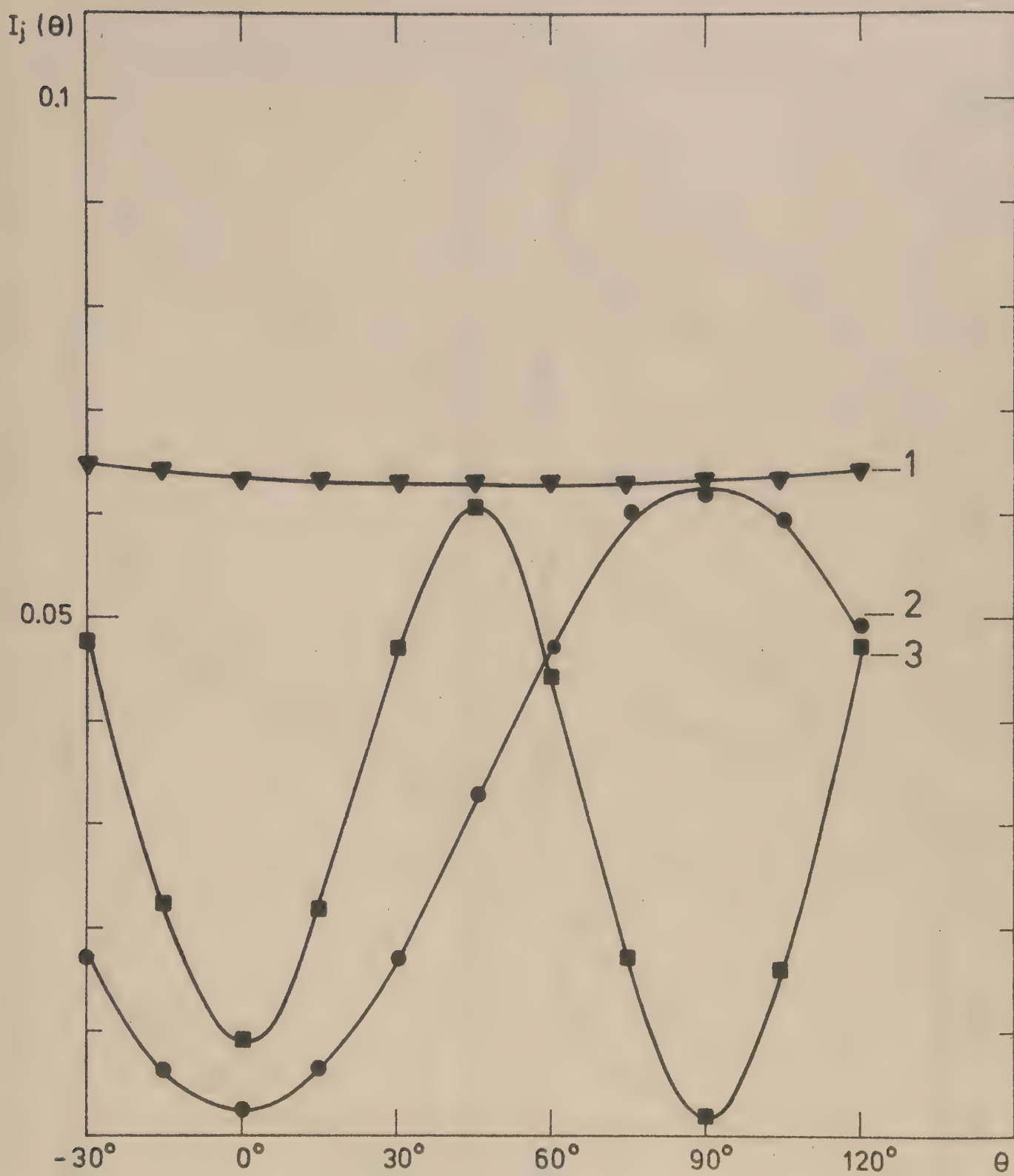
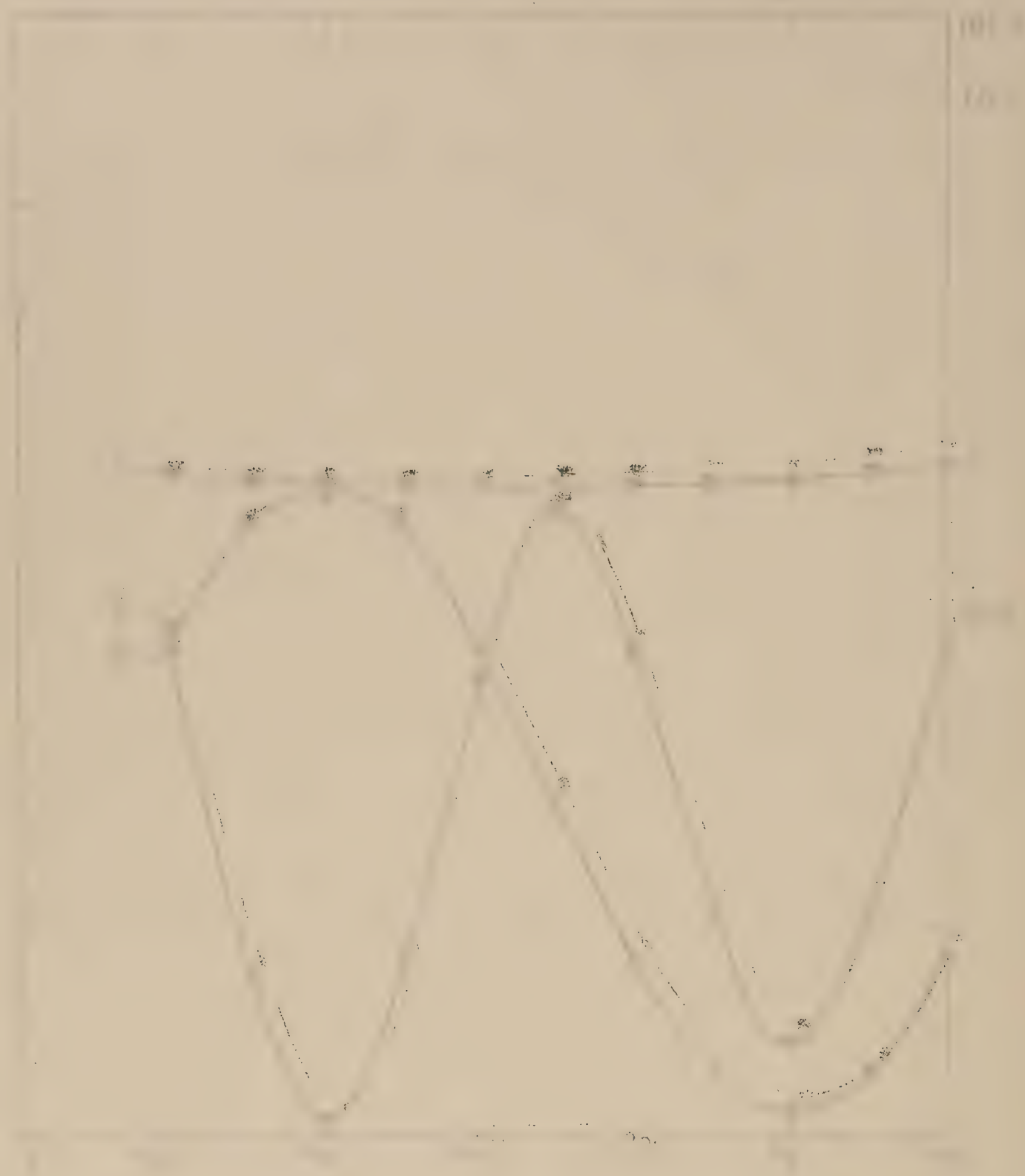


Fig. 4



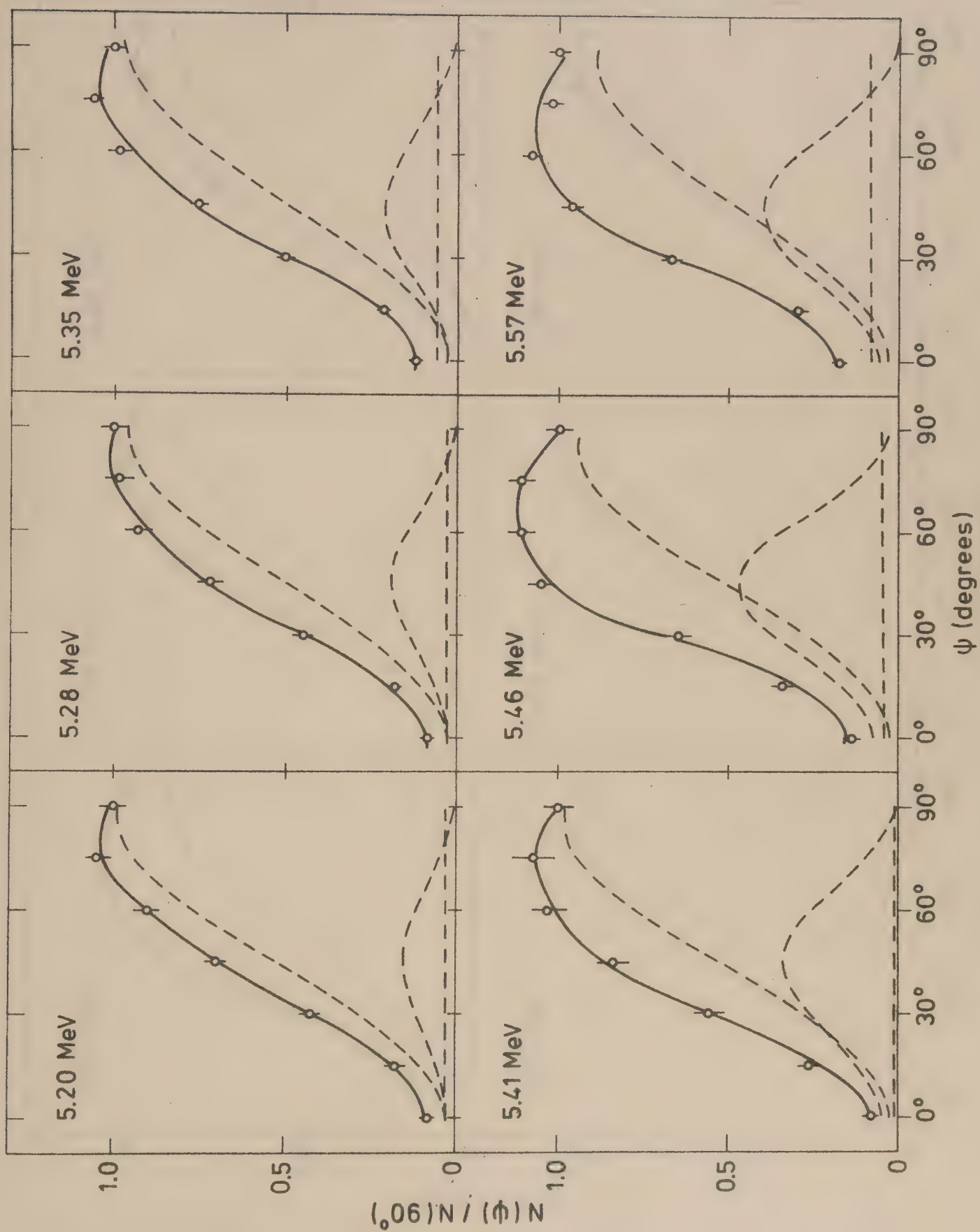


Fig. 5

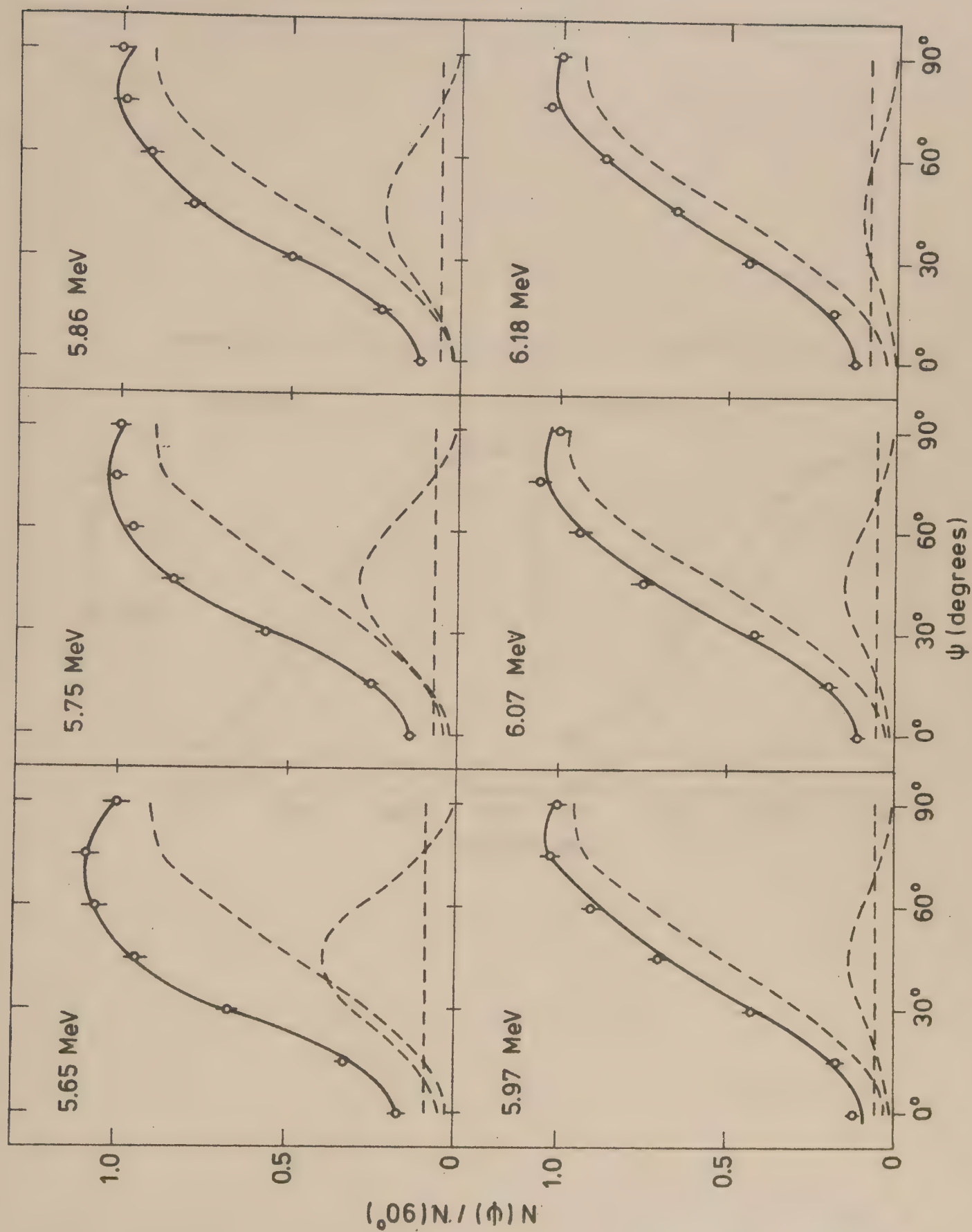


Fig. 6

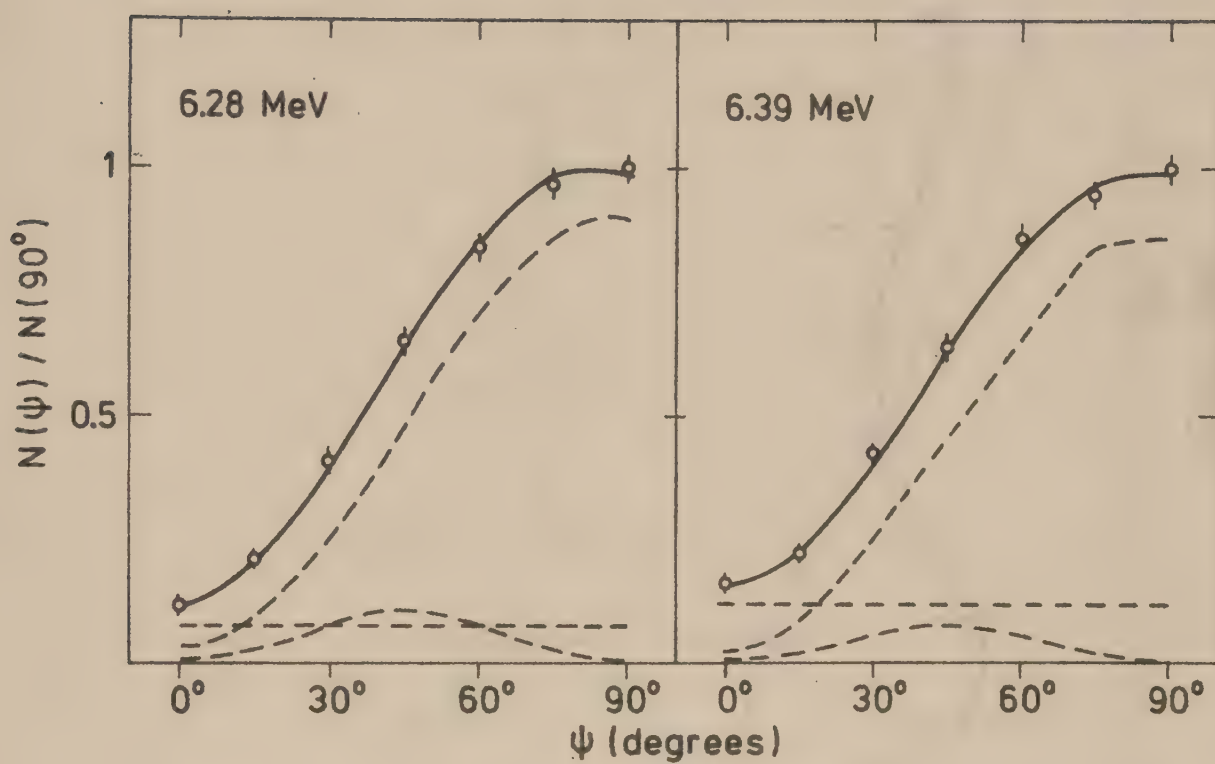
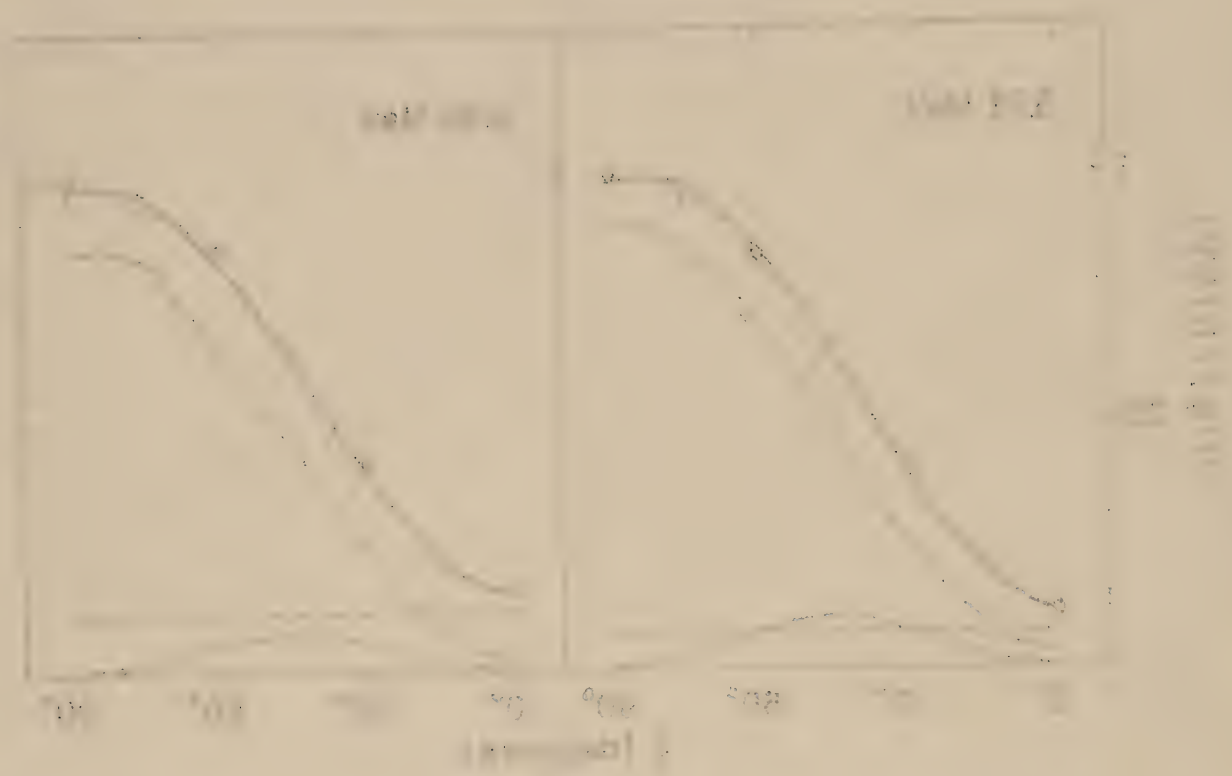


Fig. 7



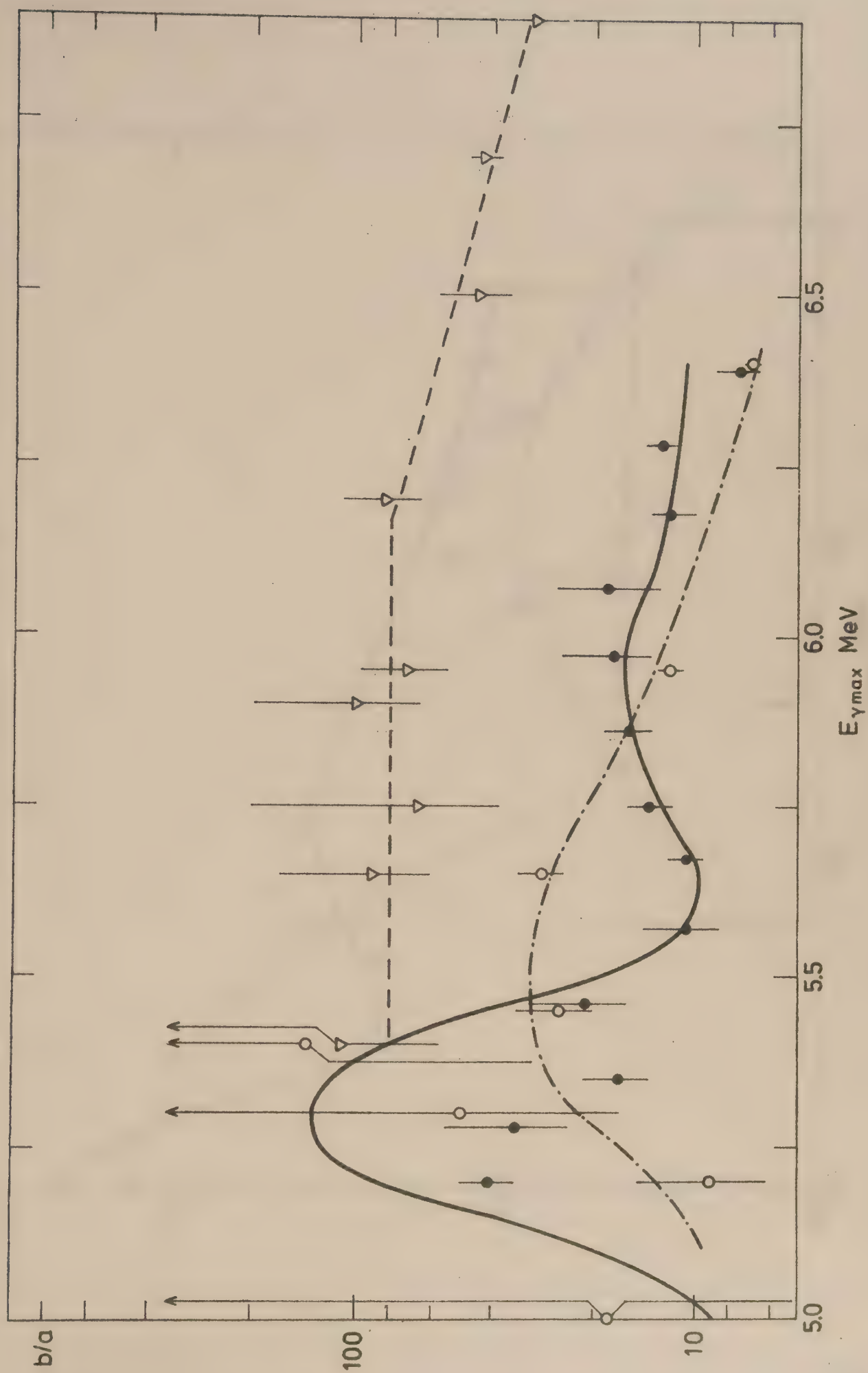


Fig. 8



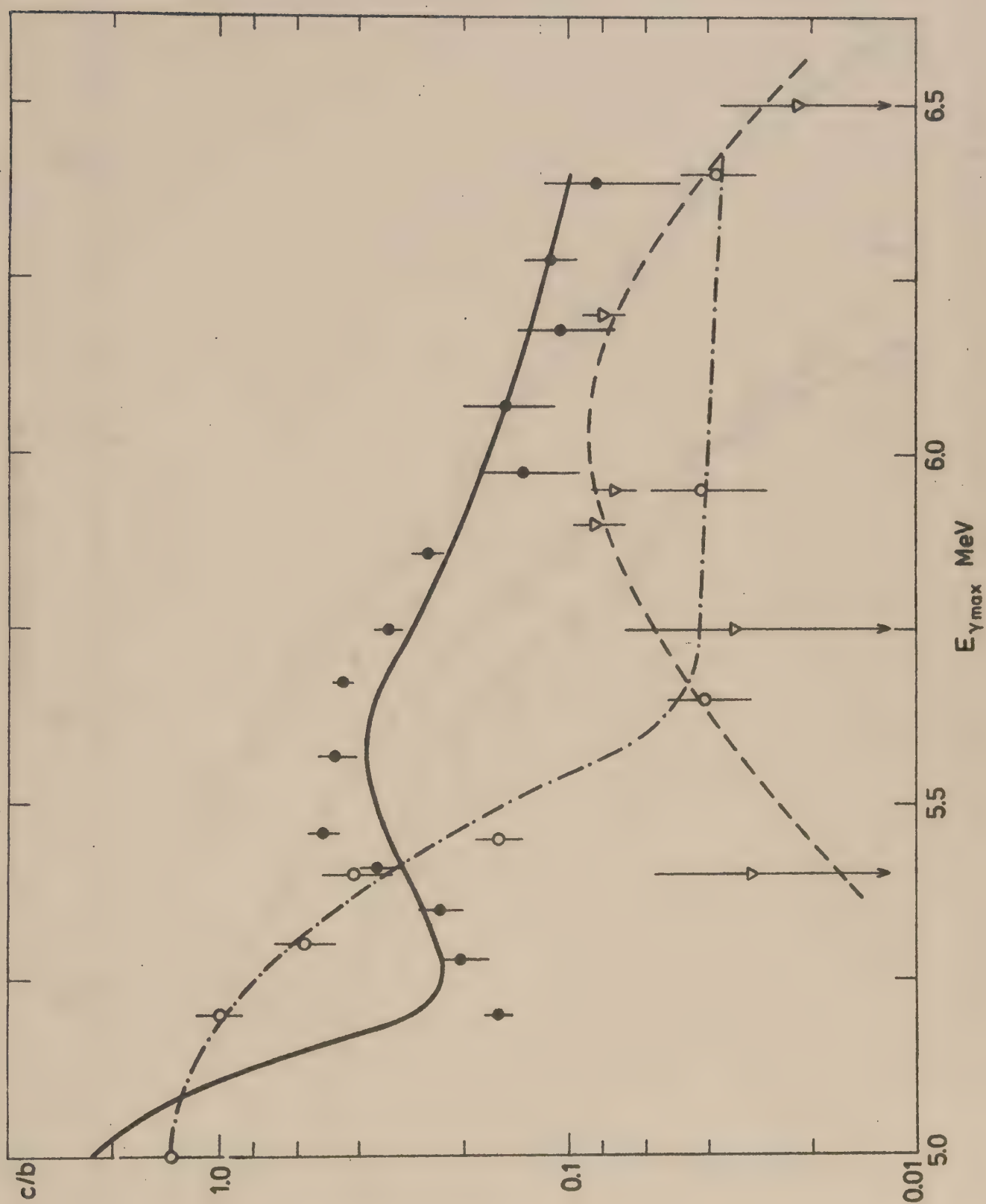


Fig. 9



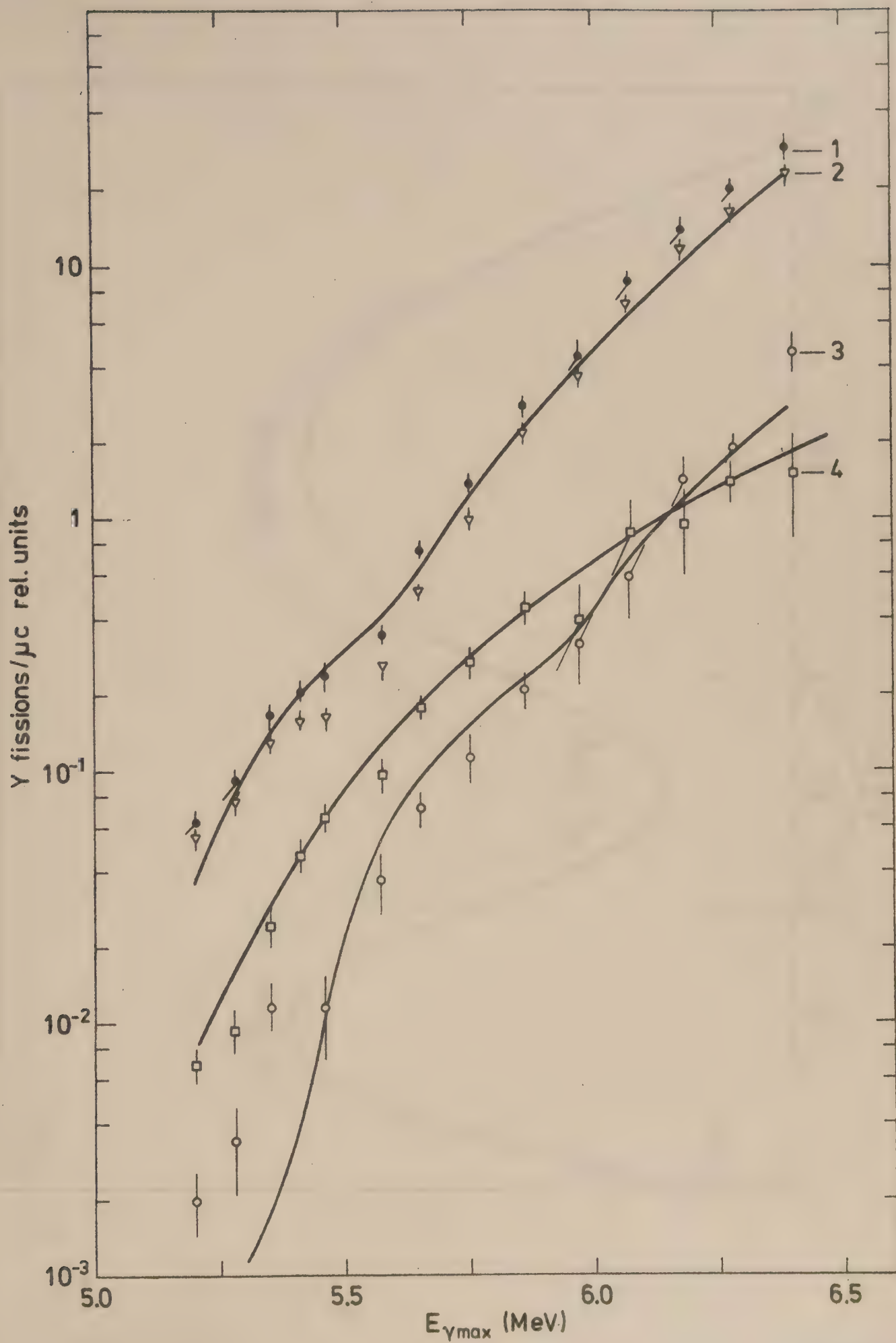


Fig. 10



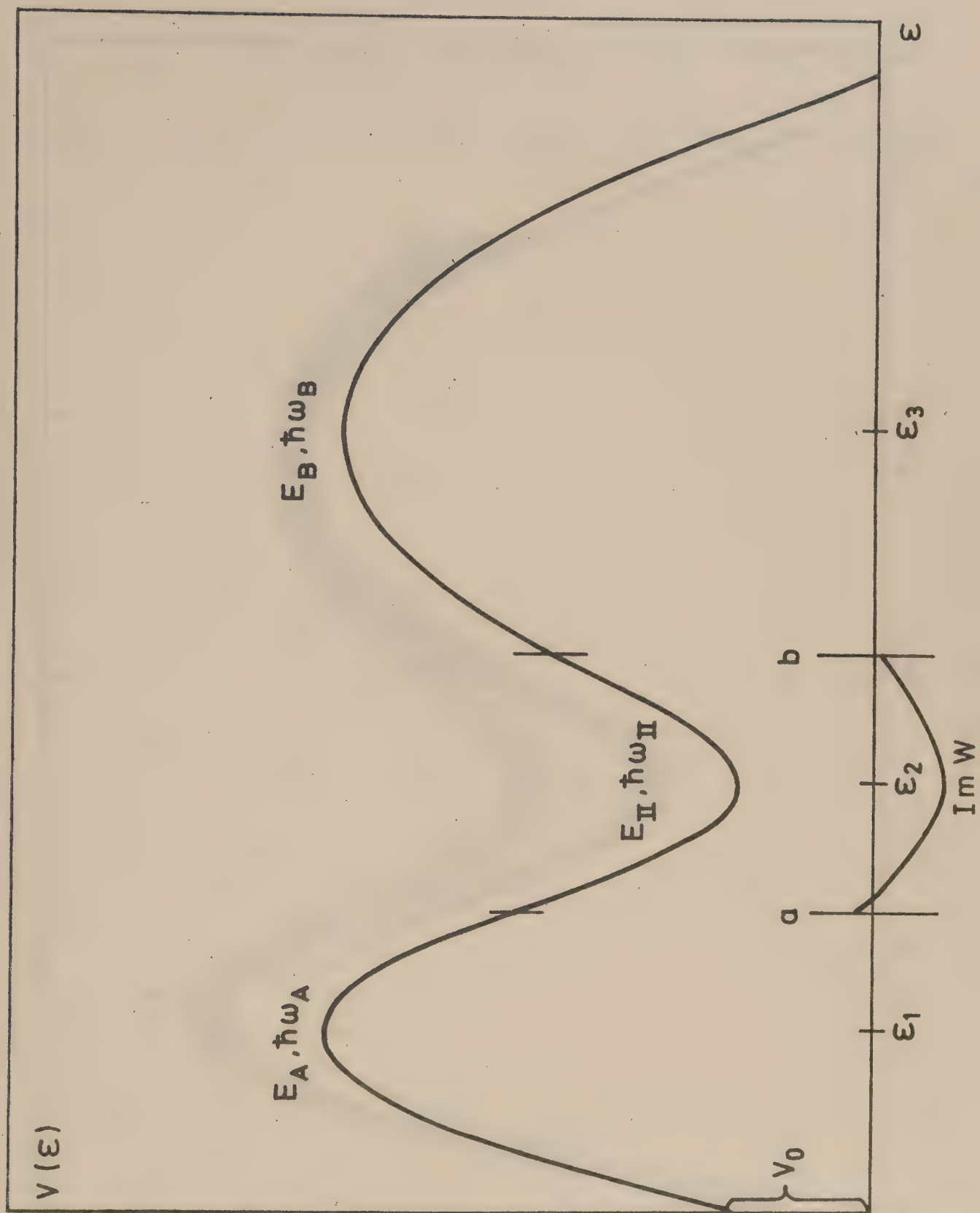


Fig. 11

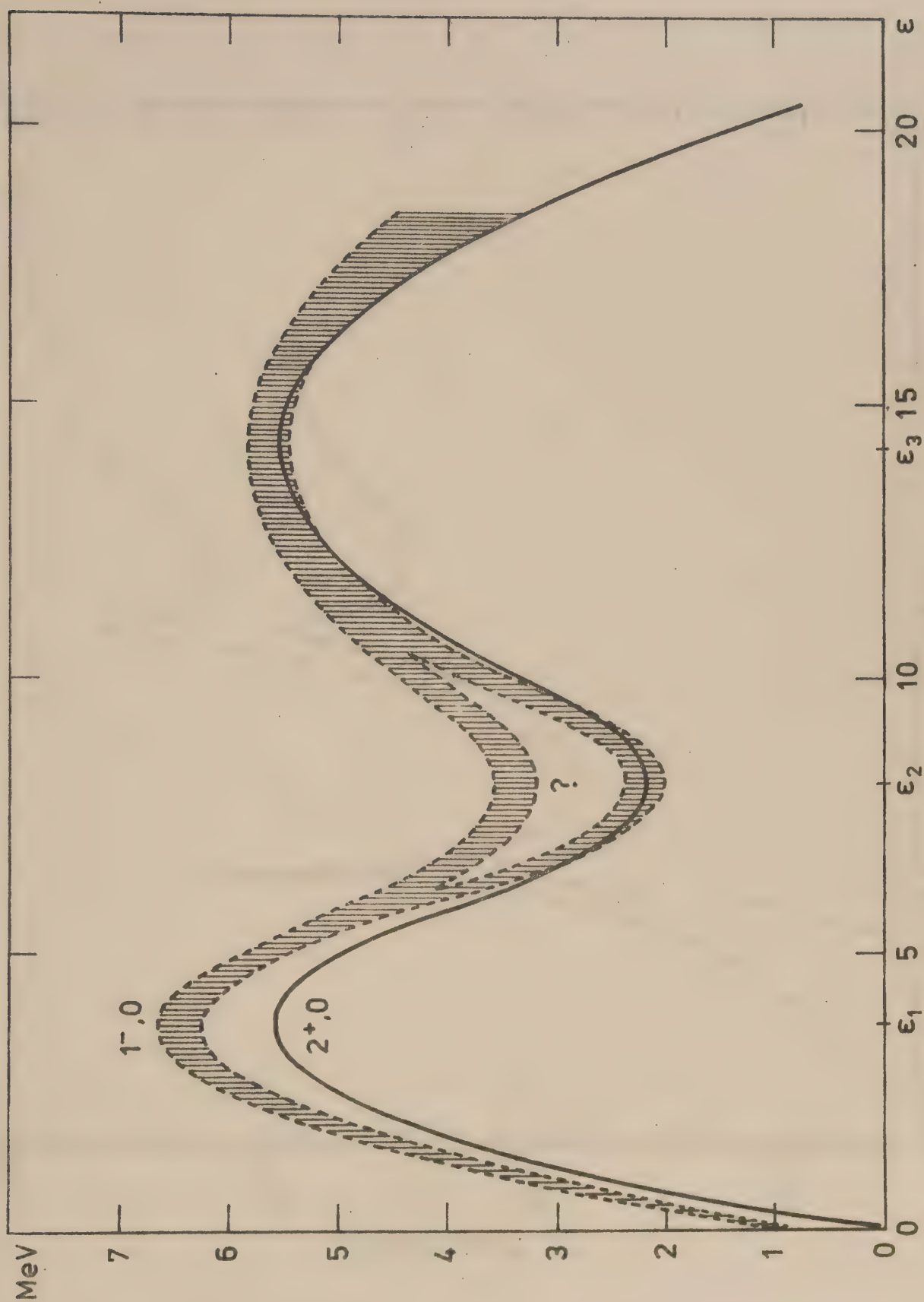


Fig. 12

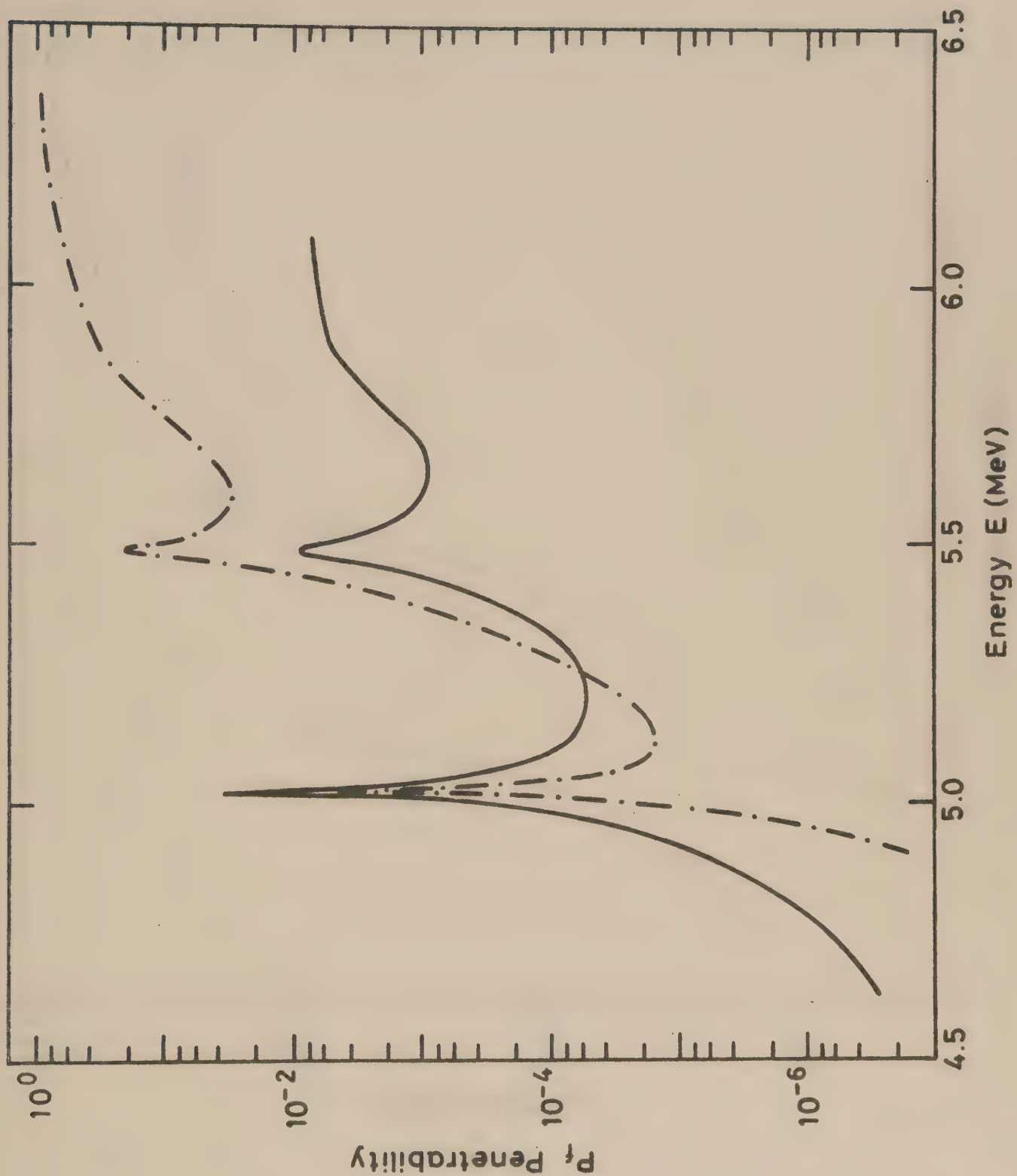


Fig. 13

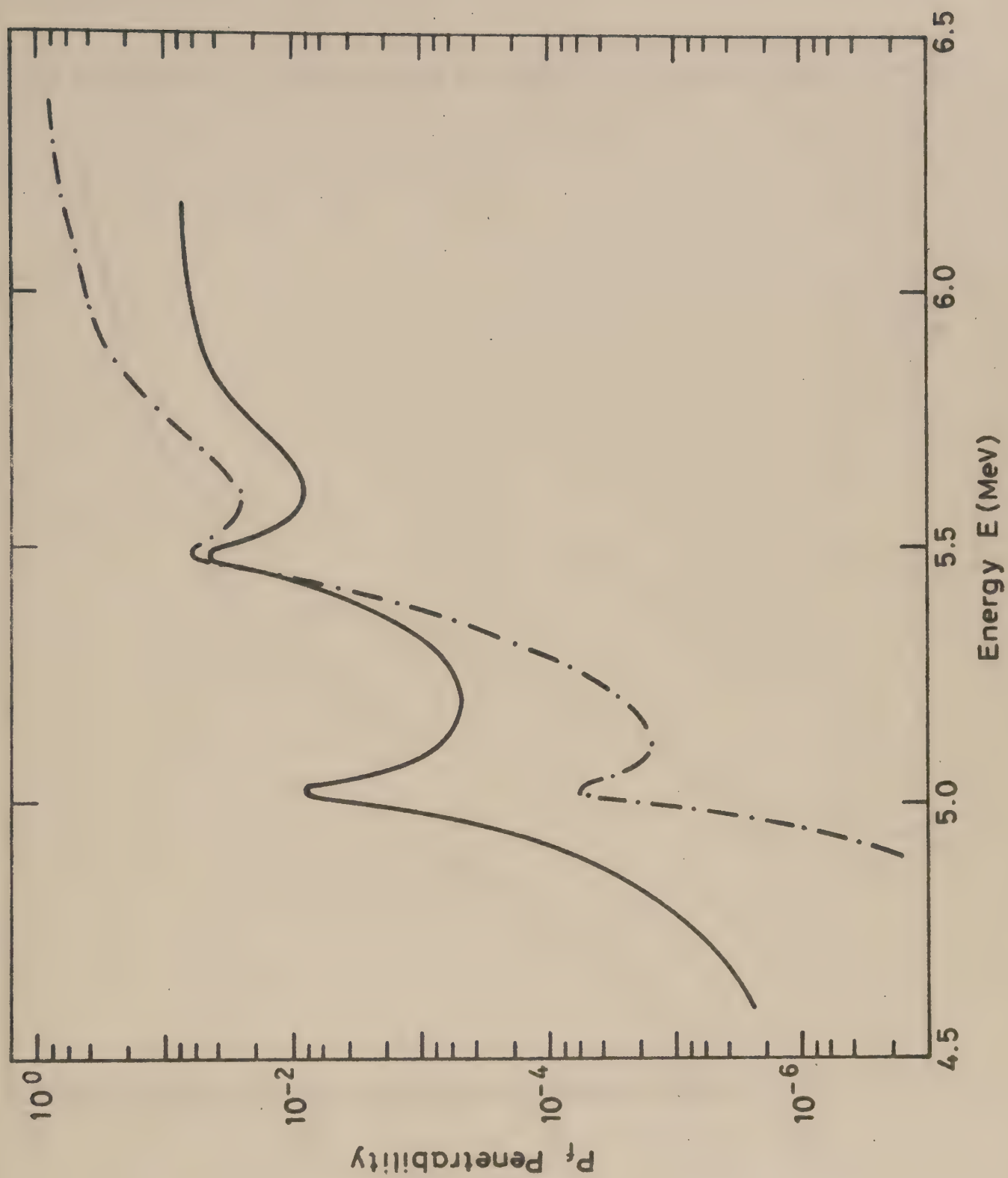
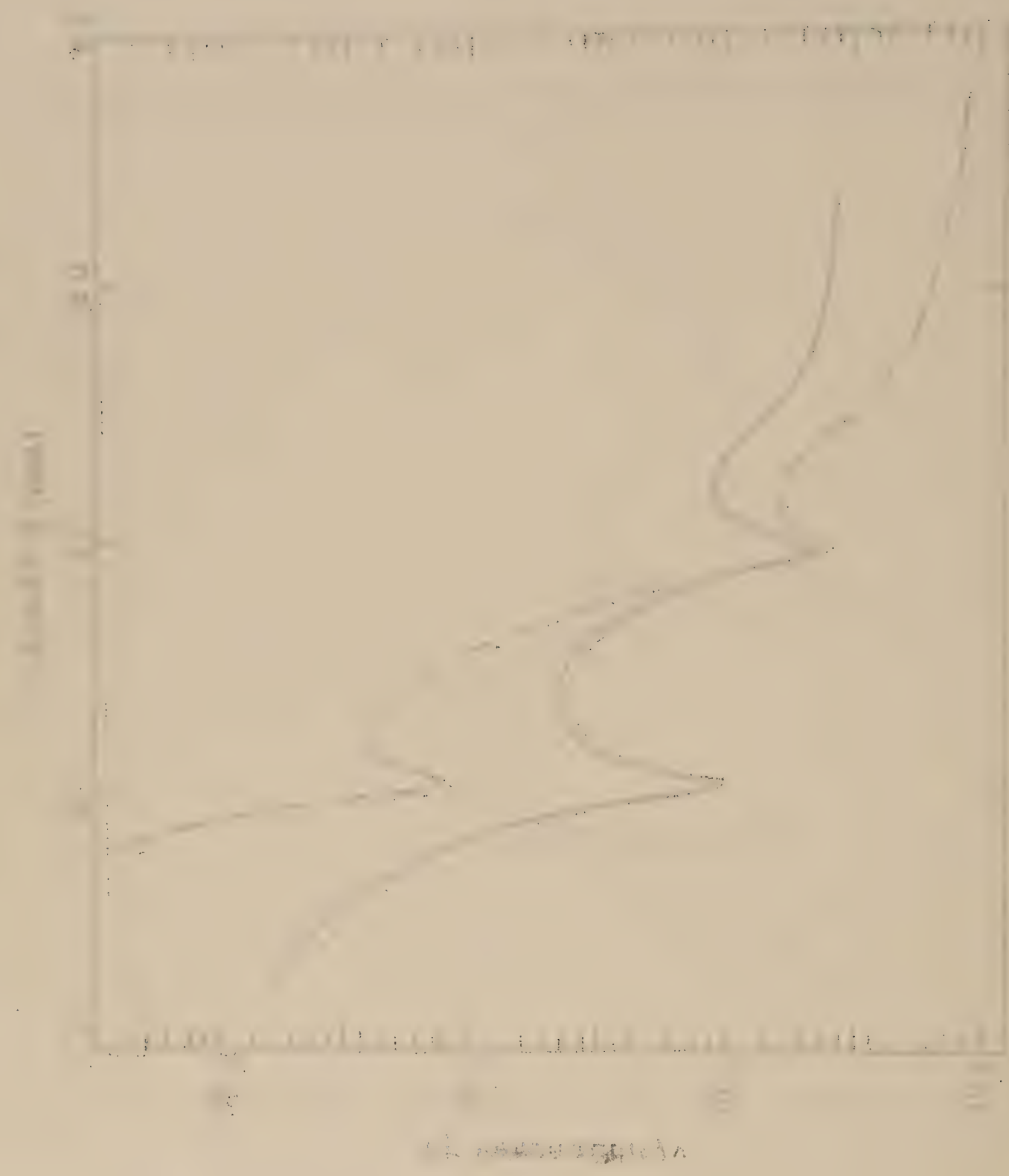


Fig. 14



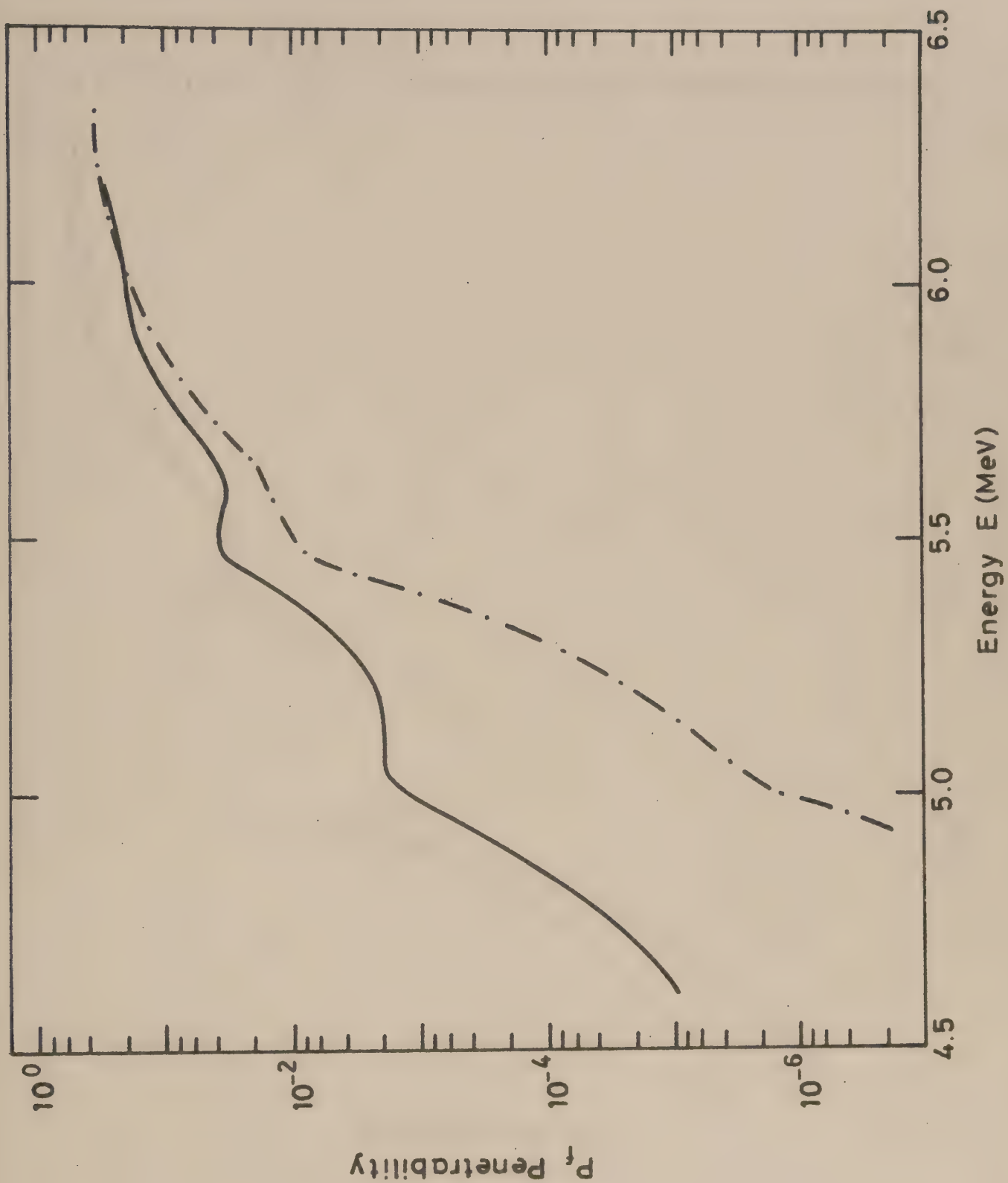


Fig. 15

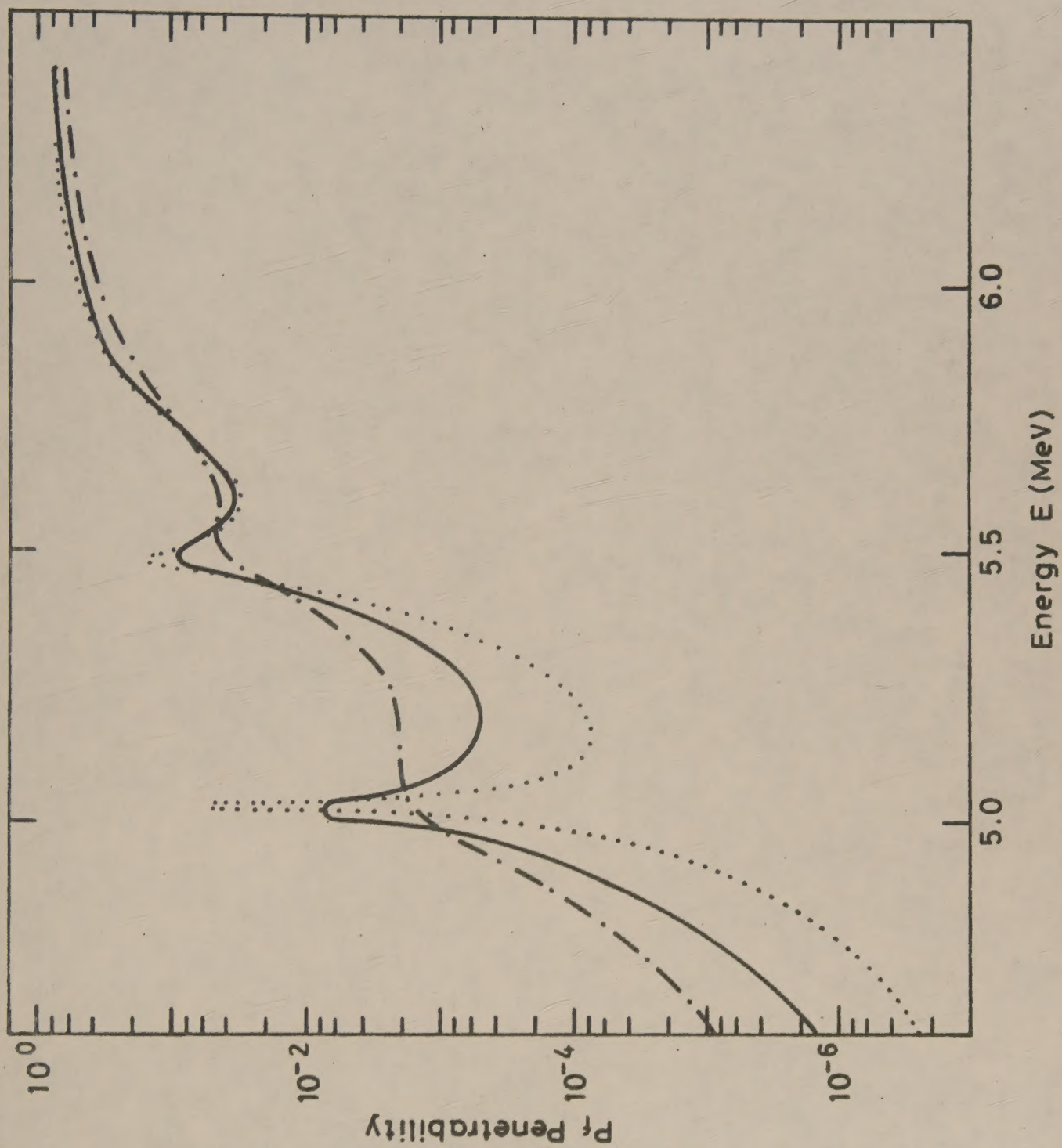


Fig. 16

